Concentrated Stopping Set Design for Coded Merkle Tree: Improving Security Against Data Availability Attacks in Blockchain Systems

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Electrical and Computer Engineering
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Non-Volatile Memories Workshop
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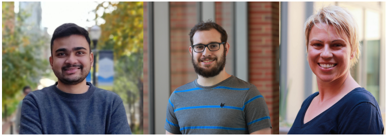

UCLA Samueli School of Engineering

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Blockchain

- Institutional Trust Systems

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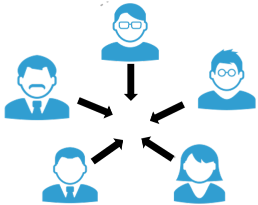

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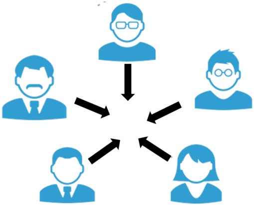
  Multiple parties collaborate on a specific task without parties trusting one another



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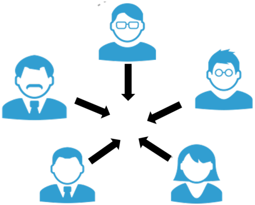

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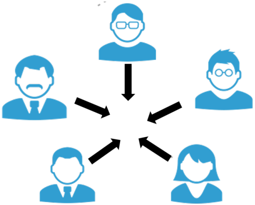

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Blockchain: Allows for decentralized trust systems

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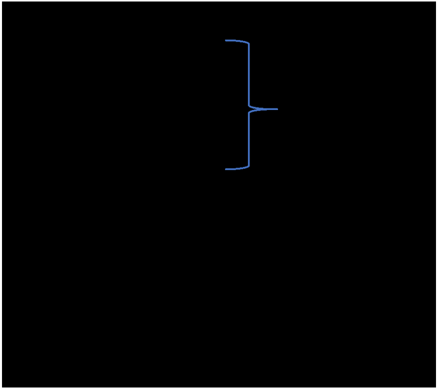
Blockchain: Allows for decentralized trust systems

  Main application of Blockchain: Currency and Finance

Blockchain Ledger



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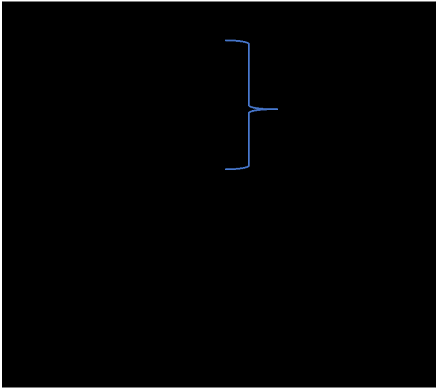

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NVM technologies like persistent memories: low latency, high reliability
Faster blockchains

Central Problem: Prohibitive Storage Overhead

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- Ethereum ledger size 650GB[2]

As of 4/28/2022, [1]https://www.blockchain.com/charts/blocks-size
[2]https://etherscan.io/chartsync/chaindefault

Central Problem: Prohibitive Storage Overhead

- Each node stores a copy of the ledger in its memory
- Prohibitive for resource limited nodes

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  !   Rely on honest Full nodes for fraud noti cation
- Full nodes send veri able fraud proofs to the light nodes to reject invalid blocks

Data Availability(DA) Attack

Systems with light nodes and a dishonest majority of full nodes are vulnerable to DA attacks [Al-Bassam '18], [Yu '19]

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$$1 \quad \frac{1}{16} \quad 1 \quad \frac{1}{15} = 0{:}87$$

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  - Probability of Light node failure using s random samples = $(1 \quad)^s$

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  What about the undecodable ratio?

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- Substructure in the Tanner Graph

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Our work: Design of specialized LDPC codes with a coupled sampling strategy to achieve a signi cantly lower probability of failure.

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  ! Prob. of failure#

Concentrated Stopping Set Design

Code Design Idea:

- Concentrate stopping sets to a small section of VNs

Concentrated Stopping Set Design

SS distribution

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Code Design Idea:

- Concentrate stopping sets to a small section of VNs
- Greedily Sample this small section of VNs

How to Concentrate Stopping Sets?

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How to design codes with concentrated cycles?
We do so by modifying the well-known Progressive Edge Growth (PEG) algorithm

PEG Algorithm

- Constructs a Tanner Graph in an edge by edge manner [Xiao '05]

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  Expand Tanner Graph in a BFS fashion

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We modify the CN selection criteria in green to concentrate cycles

Using Entropy to Concentrate Cycles

For distribution $p = (p_1; p_2; \dots; p_n)$, Entropy $H(p) = \sum_{i=1}^{n} p_i \log \frac{1}{p_i}$

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We want the cycle distributions to be concentrated

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EC (Entropy Constrained)-PEG Algorithm
For each VN v_j
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    Select CN that results in minimum entropy of resultant cycle distribution
    Update cycle distribution

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EC-PEG Algorithm

- Whenever a new edge, that creates cycles, is added to the Tanner Graph, we update the cycle counts of each VN

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$$\text{VNs}\,(v_1; v_2; :::; v_n)$$

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$$
\begin{aligned}
{}^{(6)}_{1} &= {}^{(6)}_{1} + 1 \\
{}^{(6)}_{2} &= {}^{(6)}_{2} + 1 \\
{}^{(6)}_{6} &= {}^{(6)}_{6} + 1
\end{aligned}
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- $^{(g)}_{i}$:= No. of cycles of length g that v_i is a part of, $g = 4; 6; 8$
-

$$
\begin{aligned}
^{(6)}_{1} &= {}^{(6)}_{1} + 1 \\
^{(6)}_{2} &= {}^{(6)}_{2} + 1 \\
^{(6)}_{7} &= {}^{(6)}_{7} + 1
\end{aligned}
$$

EC-PEG Algorithm

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$$^{(6)}_{1} = \ ^{(6)}_{1} + 1$$
$$^{(6)}_{3} = \ ^{(6)}_{3} + 1$$
$$^{(6)}_{6} = \ ^{(6)}_{6} + 1$$

EC-PEG Algorithm: CN Selection Procedure

Candidate CNs : c_8, c_9, c_{10}

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EC-PEG Algorithm: CN Selection Procedure

- $({}^{(4)}_{1};\ldots;{}^{(4)}_{n});({}^{(6)}_{1};\ldots;{}^{(6)}_{n});({}^{(8)}_{1};\ldots;{}^{(8)}_{n})$
- $({}^{(4)}_{1};\ldots;{}^{(4)}_{n});({}^{(6)}_{1};\ldots;{}^{(6)}_{n});({}^{(8)}_{1};\ldots;{}^{(8)}_{n})$
- $({}^{(4)}_{1};\ldots;{}^{(4)}_{n});({}^{(6)}_{1};\ldots;{}^{(6)}_{n});({}^{(8)}_{1};\ldots;{}^{(8)}_{n})$

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- $(\ _1^{(4)}; \ldots; \ _n^{(4)}); (\ _1^{(6)}; \ldots; \ _n^{(6)}); (\ _1^{(8)}; \ldots; \ _n^{(8)})$
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$\underbrace{(\ _1^{(g)}; \ldots; \ _n^{(g)})}_{\text{cycle counts}}$

EC-PEG Algorithm: CN Selection Procedure

- $({}^{(4)}_{1}, \dots, {}^{(4)}_{n}); ({}^{(6)}_{1}, \dots, {}^{(6)}_{n}); ({}^{(8)}_{1}, \dots, {}^{(8)}_{n})$
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- $({}^{(4)}_{1}, \dots, {}^{(4)}_{n}); ({}^{(6)}_{1}, \dots, {}^{(6)}_{n}); ({}^{(8)}_{1}, \dots, {}^{(8)}_{n})$

$$\underbrace{({}^{(g)}_{1}, \dots, {}^{(g)}_{n})}_{\text{cycle counts}} \rightarrow \underbrace{\left(\frac{{}^{(g)}_{1}}{\sum_{i=1}^{n} {}^{(g)}_{i}}, \dots, \frac{{}^{(g)}_{n}}{\sum_{i=1}^{n} {}^{(g)}_{i}} \right) := {}^{(g)}}_{\text{normalized counts}}$$

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- $({}_1^{(4)}; \ldots; {}_n^{(4)}); ({}_1^{(6)}; \ldots; {}_n^{(6)}); ({}_1^{(8)}; \ldots; {}_n^{(8)})$
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EC-PEG Algorithm: CN Selection Procedure

- $({}^{(4)}_{1} ; : : : ; {}^{(4)}_{n}); ({}^{(6)}_{1} ; : : : ; {}^{(6)}_{n}); ({}^{(8)}_{1} ; : : : ; {}^{(8)}_{n}) ! \; H \left(\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3} \right)$
- $({}^{(4)}_{1} ; : : : ; {}^{(4)}_{n}); ({}^{(6)}_{1} ; : : : ; {}^{(6)}_{n}); ({}^{(8)}_{1} ; : : : ; {}^{(8)}_{n})$
- $({}^{(4)}_{1} ; : : : ; {}^{(4)}_{n}); ({}^{(6)}_{1} ; : : : ; {}^{(6)}_{n}); ({}^{(8)}_{1} ; : : : ; {}^{(8)}_{n})$

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- $({}_1^{(4)}, \ldots, {}_n^{(4)}); ({}_1^{(6)}, \ldots, {}_n^{(6)}); ({}_1^{(8)}, \ldots, {}_n^{(8)})\,!\; H\left(\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3}\right)$
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- $({}_{1}^{(4)} ; : : : ; {}_{n}^{(4)}) ; ({}_{1}^{(6)} ; : : : ; {}_{n}^{(6)}) ; ({}_{1}^{(8)} ; : : : ; {}_{n}^{(8)}) ! \ H \ (\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3})$
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- $({}_1^{(4)} ; \ldots ; {}_n^{(4)}); ({}_1^{(6)} ; \ldots ; {}_n^{(6)}); ({}_1^{(8)} ; \ldots ; {}_n^{(8)}) ! \; H \; \left(\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3} \right)$
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CN selection procedure:

EC-PEG Algorithm: CN Selection Procedure

- $\left({}^{(4)}_{1} ;:::; {}^{(4)}_{n} \right); \left({}^{(6)}_{1} ;:::; {}^{(6)}_{n} \right); \left({}^{(8)}_{1} ;:::; {}^{(8)}_{n} \right)! \, H \left(\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3} \right)$
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- $\left({}^{(4)}_{1} ;:::; {}^{(4)}_{n} \right); \left({}^{(6)}_{1} ;:::; {}^{(6)}_{n} \right); \left({}^{(8)}_{1} ;:::; {}^{(8)}_{n} \right)! \, H \left(\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3} \right)$

CN selection procedure:

Select CN that results in minimum $H \left(\frac{{}^{(4)} + {}^{(6)} + {}^{(8)}}{3} \right)$

EC-PEG Algorithm: CN Selection Procedure

- $({}_1^{(4)};:::;{}_n^{(4)});({}_1^{(6)};:::;{}_n^{(6)});({}_1^{(8)};:::;{}_n^{(8)})!\,H\left(\frac{{}^{(4)}+{}^{(6)}+{}^{(8)}}{3}\right)$
- $({}_1^{(4)};:::;{}_n^{(4)});({}_1^{(6)};:::;{}_n^{(6)});({}_1^{(8)};:::;{}_n^{(8)})!\,H\left(\frac{{}^{(4)}+{}^{(6)}+{}^{(8)}}{3}\right)$
- $({}_1^{(4)};:::;{}_n^{(4)});({}_1^{(6)};:::;{}_n^{(6)});({}_1^{(8)};:::;{}_n^{(8)})!\,H\left(\frac{{}^{(4)}+{}^{(6)}+{}^{(8)}}{3}\right)$

CN selection procedure:
Select CN that results in minimum $H\left(\frac{{}^{(4)}+{}^{(6)}+{}^{(8)}}{3}\right)$

Note:

- Minimizing the entropy of joint cycle counts ensures that all cycle distributions are concentrated towards the same set of VNs

Sampling Strategy

- Our sampling strategy greedily samples VNs that are part of a large number of cycles

```
g = smallest cycle length in Tanner Graph G
While sample set size < s
    v = VN that is part of largest no. of
    cycles of length g in G
    sample set = sample set ∪ v
    remove v and all incident edges from G
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      g = g + 2
```

Simulation Results

- Code parameters: Code length $= 100$, VN degree $= 4$, Rate $= \frac{1}{2}$, girth $= 6$.

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VN index

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Simulation Results

- Code parameters: Code length = 100, VN degree = 4, Rate = $\frac{1}{2}$, girth = 6.

Fraction of cycles touched

VN index

- VN indices arranged in decreasing order of cycle 6 fractions
- Cycle 6 and cycle 8 concentrated towards same set of VNs

Simulation Results

Fraction of SSs of size 11, 12 touched by di erent VNs

Simulation Results

Fraction of SSs of size 11, 12 touched by di erent VNs

SSs of size 11

fraction of SSs touched

VN index

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Simulation Results

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SSs of size 11

fraction of SSs touched

VN index

SSs of size 12

fraction of SSs touched

VN index

- VN indices arranged in decreasing order of cycle 6 fractions

Simulation Results

Fraction of SSs of size 11, 12 touched by different VNs

SSs of size 11

fraction of SSs touched

VN index

SSs of size 12

fraction of SSs touched

VN index

- VN indices arranged in decreasing order of cycle 6 fractions
- SSs are concentrated towards the same set of VNs as the cycles

Simulation Results

Probability of failure for a stopping set of size

Simulation Results

Probability of failure for a stopping set of size

RS: Random Sampling

Prob. of failure

No of Sampless

Simulation Results

Probability of failure for a stopping set of size

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Simulation Results

Probability of failure for a stopping set of size

RS: Random Sampling
GS: Greedy Sampling

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No of Sampless

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- Concentrated LDPC codes with Greedy sampling improve the probability of failure

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- Concentrated LDPC codes with Greedy sampling improve the probability of failure

Note that the probability of failure depends on the fraction of stopping sets touched (by greedy sampling) and not the actual number.

Incorrect Coding Proof Size

- Depends on the maximum check node degree

Rate	Code length	VN degree	Ensemble [Yu '19]	PEG	EC-PEG
$\frac{1}{2}$	100	4	16	9	11
	200	4	16	9	15
$\frac{1}{4}$	100	4	8	7	10
	200	4	8	6	9

Table: Maximum CN degree for di erent codes.

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Table: Maximum CN degree for di erent codes.

- Concentrated LDPC codes do not sacri ce on the incorrect coding proof size

Conclusion and Extentions

- Summary:
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- Summary:
    - We provided a specialized code construction technique to concentrat stopping sets in LDPC codes
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- Extensions (Mitra '21):
    - Considered stronger adversary models that can selectively pick a stopping set that has a lower probability of being sampled to hide instead of randomly
    - Provided optimal sampling strategies and associated coupled LDPC code construction to improve the security against such strong adversaries for a given sample complexity

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