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Trading Partially Stuck Cells with Errors

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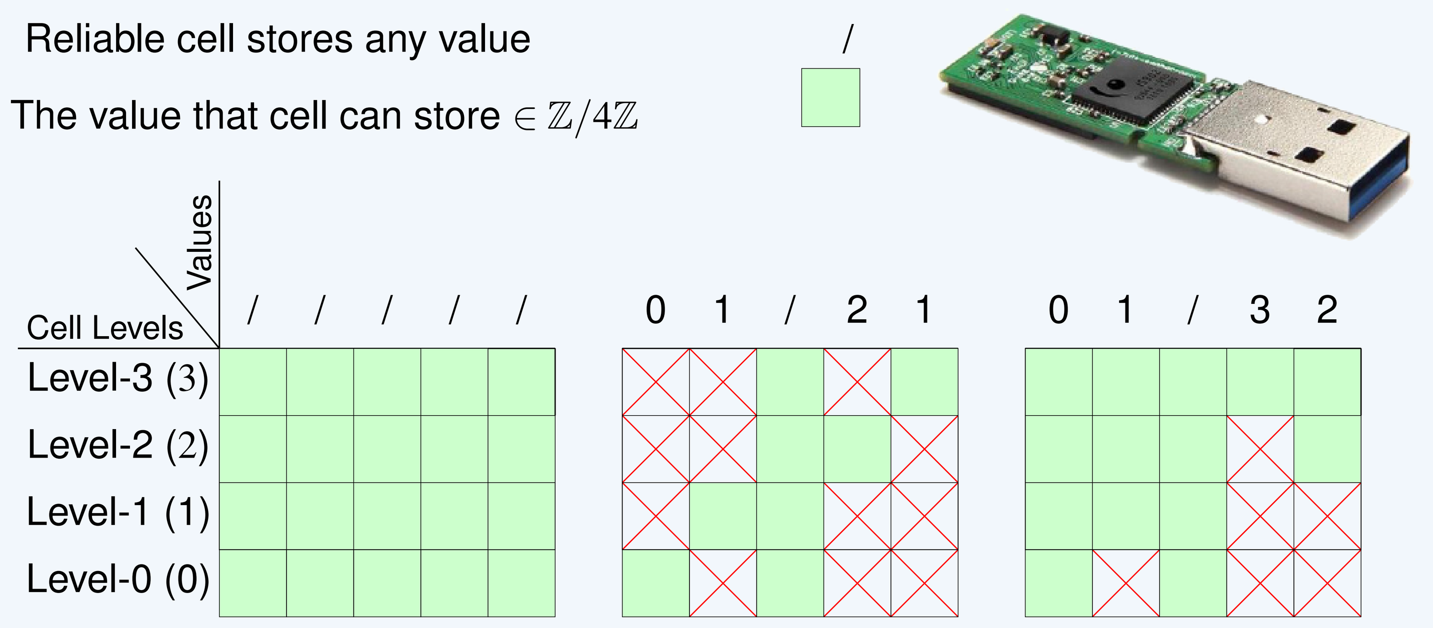
Introduction

The dominance of non-volatile memories and PCMs (phase change memories) as memory solutions for variety of applications have become obvious due to their advantages as permanent storage devices [1].

Problem Description: PCMs may face failures in changing their states, in turn, its cells hold only one phase and they become **stuck** (**defective**). On the other hand, random errors may occur on these defective memories.

Solution: A mechanism called **masking** is used to determine a word whose entries coincide with writable levels at the (partially) stuck cells.



(A) Reliable Cells (B) Stuck Cells (C) Partially Stuck Cells

Previous works: In [2], the author considered the problem of masking fully stuck cells together with error correction. The error-free case with partially stuck cells has been considered in [3] in which improvements on the redundancy necessary for masking compared to [2] are achieved.

Our Contribution: In our code constructions in [4], we consider the problem of combined error correction and masking of partially stuck cells, and we reduce the redundancy necessary for masking, similar to the results in [3] and even reduce further compared to [3, Construction 5].

Focus: We propose techniques of using part of the error-correcting capability to mask more partially stuck cells. This observation was also made in [1,Theorem 1].

Definitions

(Σ, t)-PSMC: For $\Sigma \subset \mathbb{F}_q^n$ and non-negative integer t, a q-ary (Σ, t)-*partially-stuck-at-masking code* $\mathcal{C}$ of length n and size M is a coding scheme consisting of a message set $\mathcal{M}$ of size M, an encoder $\mathcal{E}$ and a decoder $\mathcal{D}$. If $\Sigma = \{\mathbf{s} \in \{0,1\}^n \mid wt(\mathbf{s}) \leq u\}$, we say q-ary $(u,1,t)$ PSMC.

Proposition 1 [4, Proposition 3]

If there is an $(n,M)_q(u,1,t)$ PSMC, then for any j with $0 \leq j \leq t$, there is an $(n,M)_q(u+j,1,t-j)$ PSMC.

Theorem 1 (Partial Masking PSMC) [4, Theorem 6]

Let $\Sigma \subset \mathbb{F}_q^n$, and assume that there exists an $(n,M)_q(\Sigma,t)$ PSMC $\mathcal{C}$. For any $j \in [t]$, there exists an $(n,M)_q(\Sigma^{(j)},t-j)$ PSMC $\mathcal{C}_j$, where

$$\Sigma^{(j)} = \left\{\mathbf{s}' \in \mathbb{F}_q^n \mid \exists \mathbf{s} \in \Sigma \left[d(\mathbf{s},\mathbf{s}') \leq j \text{ and } \mathbf{s}' \geq \mathbf{s}\right]\right\}.$$

Proof of Theorem 1

Let the encoder $\mathcal{E}_j$ and the decoder $\mathcal{D}_j$ for $\mathcal{C}_j$ be Algorithm 1 and Algorithm 2, respectively. By definition, $\mathbf{c}' \geq \mathbf{s}'$. Moreover, if $s_i = s_i'$, then $c_i \geq s_i = s_i'$, so $c_i = c_i'$. As a result, $d(\mathbf{c},\mathbf{c}') \leq j$.

In Algorithm 2, the decoder $\mathcal{D}$ of $\mathcal{C}$ is directly used for decoding $\mathcal{C}_j$. As $\mathbf{y} \geq \mathbf{s}'$, surely $\mathbf{y} \geq \mathbf{s}$. Moreover, we can write $\mathbf{y} = \mathbf{c} + (\mathbf{c}' - \mathbf{c} + \mathbf{e})$. As shown above, wt$(\mathbf{c}' - \mathbf{c}) \leq j$, and so $wt(\mathbf{c} - \mathbf{c}' + \mathbf{e}) \leq t$. As a consequence, $\mathcal{D}(\mathbf{y}) = \mathbf{m}$.

Algorithm 1 - Encoder $\mathcal{E}_j$

Input: $(\mathbf{m},\mathbf{s}') \in \mathcal{M} \times \Sigma^{(j)}$.

1. Determine $\mathbf{s} \in \Sigma$ such that $d(\mathbf{s},\mathbf{s}') \leq j$ and $\mathbf{s}' \geq \mathbf{s}$.
2. Let $\mathbf{c} = \mathcal{E}(\mathbf{m},\mathbf{s})$.
3. Define $\mathbf{c}' = \mathcal{E}_j'(\mathbf{m},\mathbf{s}')$ as $c_i' = \max(c_i, s_i')$ for $i \in [n]$.

Output: Codeword $\mathbf{c}' \in \mathbb{F}_q^n$.

Algorithm 2 - Decoder $\mathcal{D}_j$

Input: Retrieved $\mathbf{y} = \mathbf{c}' + \mathbf{e}$ where $wt(\mathbf{e}) \leq t - j$ and $\mathbf{y} \geq \mathbf{s}'$

1. Message $\mathbf{m} = \mathcal{D}(\mathbf{y})$

Output: Message vector $\mathbf{m}$

Lemma 1 [4, Lemma 1]

Given an $[n,k,d]_q$ code as defined in [4, Construction 3], then for any j such that $0 \leq j \leq \lfloor \frac{d-1}{2} \rfloor$, there is a 2^μ-ary $(2^{\mu-1}(d_0+j)-1, 1, \lfloor \frac{d-1}{2} \rfloor - j)$ PSMC of length n and size q^{k-l-1}.

Lemma 2 [4, Lemma 2]

Given an $[n,k,d]_q$ code containing a word of weight n, for any j with $0 \leq j \leq \lfloor \frac{d-1}{2} \rfloor$, there is a q-ary $(q-1+qj, 1, \lfloor \frac{d-1}{2} \rfloor - j)$ PSMC of length n and size q^{k-1}.

Lemma 3 (Generalization of Lemma 2) [4, Lemma 3]

Given an $[n,k,d]_q$ code containing a word of weight n. Let $0 \leq j \leq \lfloor \frac{d-1}{2} \rfloor$, and let

$$\Sigma = \left\{\mathbf{s} \in \mathbb{F}_q^n \,\middle|\, \sum_i s_i \leq q-1+qj\right\}.$$

There is a q-ary $(\Sigma, \lfloor \frac{d-1}{2} \rfloor - j)$ PSMC of length n and size q^{k-1}.

Comparisons Gilbert–Varshamov-type Bounds

For [4, Construction 3], we have derived Gilbert–Varshamov-type bound in [4, Theorem 11]. Then, we have compared the direct application of [4, Theorem 11] with the exchange of a one error correction ability with a single masking capability of a partially stuck cell by Theorem 1 that further compared to Lemma 1 and Lemma 2 as shown in Figure 1.

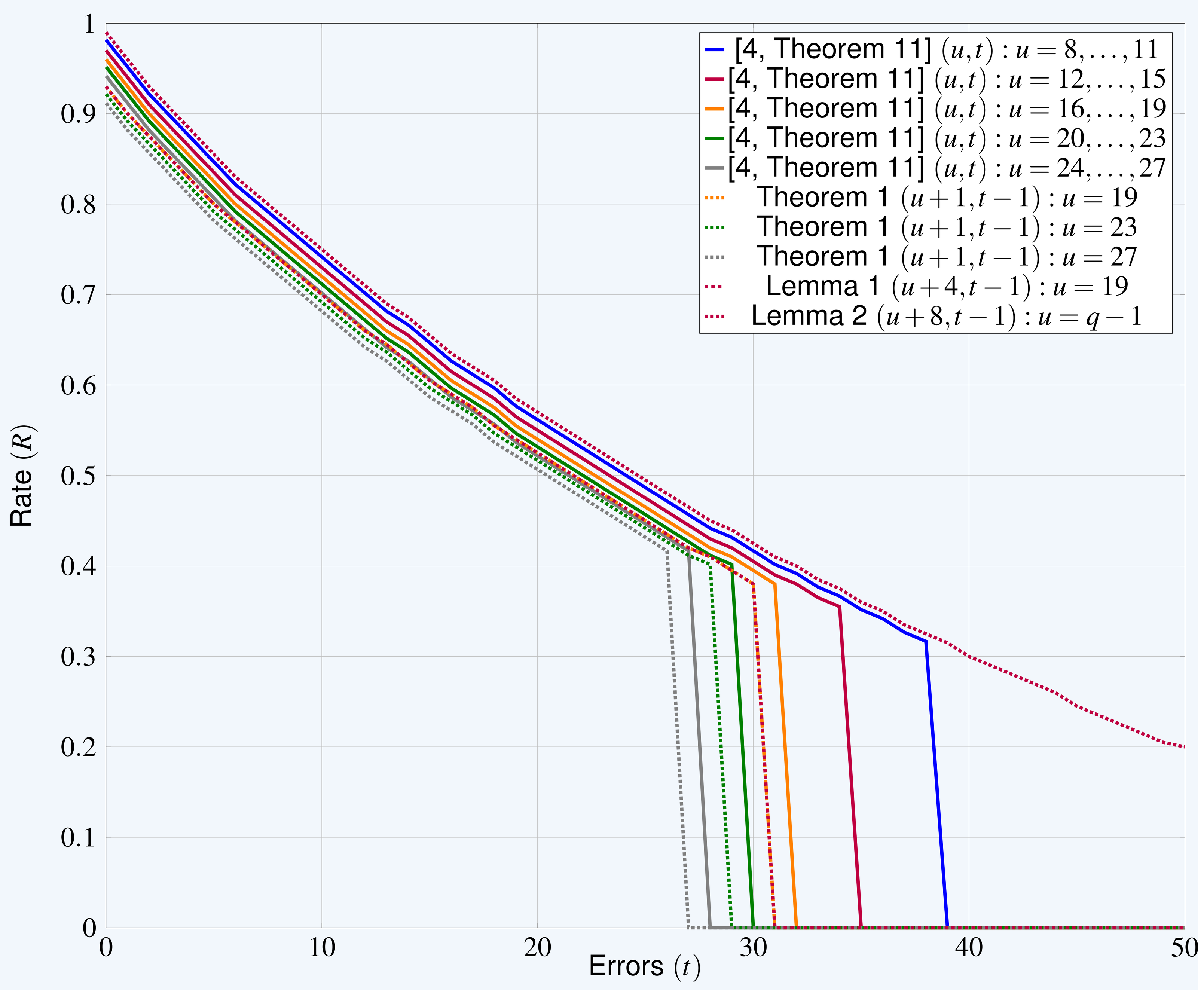


Figure 1. The achievable rates $R = \frac{1}{n}\log_q \mathcal{M}$ of GV bounds for different u, and t for $n = 200$ and $q = 2^3$ in [4, Theorem 11], Theorem 1, Lemma 1 and Lemma 2.

Conclusion

Treating some of the partially stuck cells as erroneous cells can decrease the required redundancy for some parameters using Theorem 6. Lemma 1 and Lemma 2 improve upon Theorem 6 (i.e. see the perfectly matched dotted-orange and dashed-red plots for $u = 19$).

References

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[2] C. Heegard, “Partitioned linear block codes for computer memory with ‘stuck-at’ defects,” IEEE Trans. Inf. Theory, vol. 29, no. 6, pp. 831–842, Nov. 1983.

[3] A.Wachter-Zeh and E. Yaakobi, “Codes for Partially Stuck-At Memory Cells,” IEEE Transactions on Information Theory, vol. 62, no. 2, pp. 639–654, February 2016.

[4] H. A. Kim, S. Puchinger, L. Tolhuizen, A. Wachter-Zeh, ‘Coding and Bounds for Partially Defective Memory Cells’. arXiv, 2022.

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