# Polar Codes with Local-Global Decoding

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## I. Introduction

Error correction coding schemes with local-global decoding are motivated by practical data storage applications where a balance must be achieved between low latency read access and high data reliability. As an example, consider a 4KB codeword, consisting of four 1KB subblocks, that supports a local-global decoding architecture. Local decoding can provide reliable, low-latency access to individual 1KB subblocks under good channel conditions, while global decoding can provide a "safety-net" for recovery of the entire 4KB block when local decoding fails under bad channel conditions. Recently, Ram and Cassuto have proposed such local-global decoding architectures for LDPC codes [7] and spatially coupled LDPC codes [8].

In this paper, we investigate a coupled polar code architecture that supports both local and global decoding. The coupling scheme incorporates a systematic outer polar code and a partitioned mapping of the outer codeword to semipolarized bit-channels of the inner polar codes. Error rate simulation results are presented for 2 and 4 subblocks.

## II. Background

Polar codes, proposed by Erdal Arıkan in 2009, provide a deterministic coding scheme that provably achieves the Shannon capacity of any symmetric, binary-input discrete memoryless channel under successive cancellation (SC) decoding [1]. Belief propagation (BP) decoding of polar codes, which provides soft decoder outputs with relatively low complexity, was suggested by Arıkan in [2] and has since been widely investigated. Systematic encoding of polar codes [3] is used in scenarios where it is desirable for the encoded information to appear explicitly in the codeword. In [5], Guo et al. proposed enhanced BP decoding of polar codes through concatenation of an outer code that protects bit-channels of intermediate channel quality, referred to as semipolarized bit-channels. To illustrate the idea, they considered an outer LDPC code and an outer convolutional code. Elekelesh et al. [4] extended this idea and introduced an augmented polar code construction using an auxiliary outer polar code to protect semipolarized bit-channels. They also proposed a flexible-length polar code construction that couples two polar codes of different lengths through such an auxiliary outer polar code.

The code construction and decoding scheme reported in this paper combine these ingredients into a local-global decoding architecture for coupled polar codes [10].

## III. Local-Global Polar Code Architecture

### A. Encoder

A schematic of the proposed encoder for the local-global polar code architecture is shown in Fig. 1.

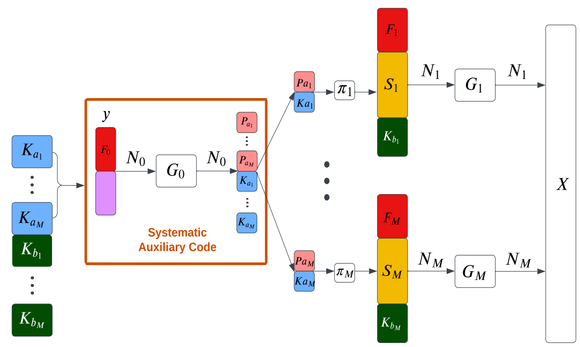


Fig. 1. Encoder for local-global polar code.

Let $[K_a, K_b] = [K_{a_1}, ..., K_{a_M}, K_{b_1}, ..., K_{b_M}]$ denote the information bits to be encoded. For purposes of illustration, we assume the sets $K_{a_i}$, $i = 1, \ldots, M$ are of equal size, and similarly for the sets $K_{b_i}$, $i = 1, \ldots, M$. The inputs to the systematic outer polar encoder are information bits $K_a$ and frozen bits $F_0$, and the length-$N_0$ output codeword is $[P_a, K_a]$. This codeword contains the information bits $K_a$, in known positions, and the parity bits $P_a$. Dividing the parity bits $P_a$ into $M$ equal-size subsets, the interleaver maps $[P_{a_i}, K_{a_i}]$ to the semipolarized bit-channels $S_i$ of the $i^{th}$ inner code. (We also assume in this illustration that the inner codes have equal lengths.) The goal of the interleaving is to decorrelate the LLRs used in the BP decoding as much as possible in the early decoder iterations.

The information bits $K_{b_i}$ and additional frozen bits $F_i$ provide the input to the good bit-channels and frozen bit-channels of the $i^{th}$ inner code, respectively. The codewords of the $M$ inner codes are concatenated to form a length-$N$ codeword that is transmitted over the channel.

The relevant code rates are as follows, where, as a shorthand notation, we use the name of a subset to represent its cardinality.

- Combined code rate: $R_{total} = \frac{K_a+K_b}{N}$
- Systematic outer polar code rate: $R_{outer} = \frac{K_a}{N_0}$
- $i^{th}$ inner polar code rate: $R_{inner,i} = \frac{K_{b_i}+S_i}{N_i}$
- $i^{th}$ subblock rate: $R_{subblock,i} = \frac{K_{b_i}+K_{a_i}}{N_i}$

### B. Decoder

The architecture proposed in Fig. 1 permits separate local decoding of the inner codes, with the option of invoking global decoding of the coupled codes when local decoding does not provide satisfactory performance. Note that this architecture also retains flexibility in the choice of inner code lengths $N_i$, if that is desired.

*1) Local decoding:* We use BP decoding with early stopping [9] for local decoding. The estimated bits on the semipolarized bit-channels must be deinterleaved to recover the information bits $K_a$.

*2) Global decoding:* The global decoding is carried out using BP decoding on the combined factor graph of the inner codes and outer code. To reduce decoding time, we check early stopping conditions for the inner codes and the outer code and stop the decoder when both are satisfied.

TABLE I
SYSTEM PARAMETER SETTINGS

| Parameters | Setting 1 | Setting 2 | Setting 3 |
|---|---|---|---|
| $R_{total}$ | 0.5 | 0.5 | 0.5 |
| $R_{outer}$ | 0.5 | 0.5 | 0.5 |
| $R_{inner}$ | 0.53125 | 0.5625 | 0.53125 |
| $M$ | 2 | 4 | 4 |
| $K_a$ | 64 | 256 | 128 |
| $K_b$ | 960 | 1792 | 1920 |
| $S_i, i \geq 1$ | 64 | 128 | 64 |
| $N_0$ | 128 | 512 | 256 |
| $N_i, i \geq 1$ | 1024 | 1024 | 1024 |
| $Max\ iteration$ | 200 | 200 | 200 |
| $Early\ stop$ | ✓ | ✓ | ✓ |

### C. Local-Global Decoding Simulation Results

BER and FER results for local-global decoding with 2 and 4 subblocks, each corresponding to an inner code of length 1024 bits, are shown in Fig. 2 and Fig. 3, respectively. The parameters settings are shown as Setting 1 and Setting 2 in Table I. Each figure shows the performance of a typical subblock under local decoding, along with that of a conventional polar code of length 1024 bits under BP decoding. Also shown is the performance of the full codeword under global decoding, along with the performance of a conventional polar code of length equal to that of the full codeword under BP decoding. Simulation results (not shown here) confirmed that the BERs and FERs of the subblocks under local and global decoding are essentially identical and that subblock failures appear to be independent. A comparison of settings 2 and 3 (not shown) demonstrates that trade-offs between local and global decoding performance can be achieved by adjusting system parameters.


## ACKNOWLEDGMENT

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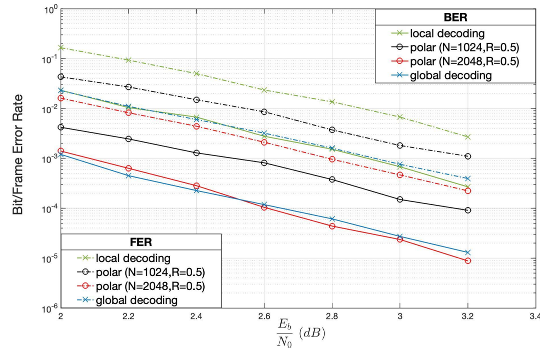


Fig. 2. Local-global decoding with 2 subblocks.

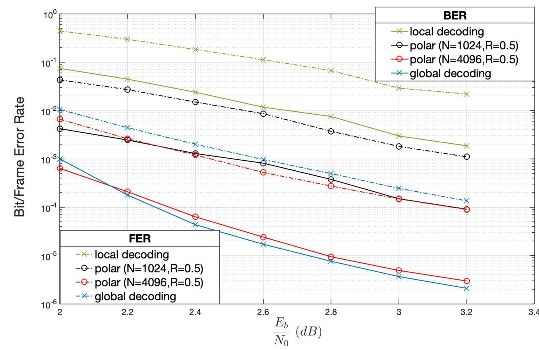


Fig. 3. Local-global decoding with 4 subblocks.

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