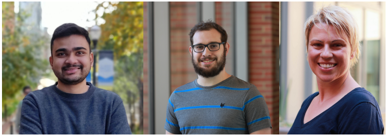
Concentrated Stopping Set Design for Coded Merkle Tree: Improving Security Against Data Availability Attacks in Blockchain Systems

Debarnab Mitra, Lev Tauz, and Lara Dolecek

Electrical and Computer Engineering
University of California, Los Angeles

Non-Volatile Memories Workshop
May 10, 2022



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Blockchain

- Institutional Trust Systems

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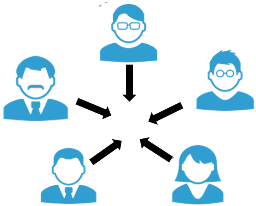

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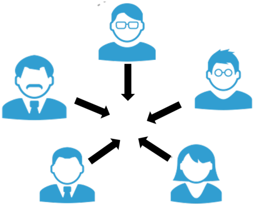
  - Multiple parties collaborate on a specific task without parties trusting one another



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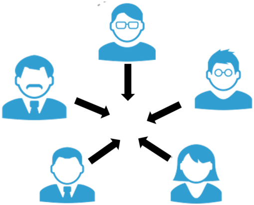

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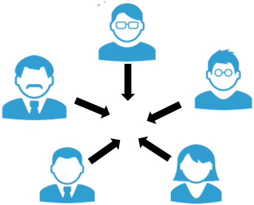

☐ Blockchain: Allows for decentralized trust systems

Blockchain

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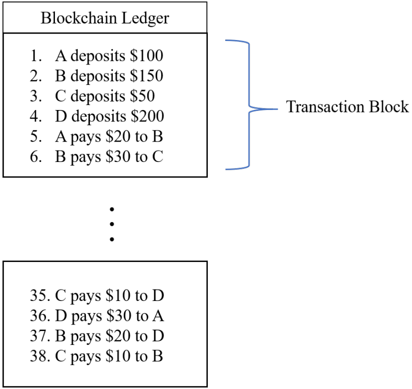
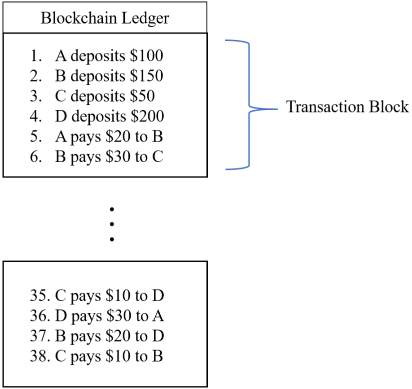
- Blockchain: Allows for decentralized trust systems
  - Main application of Blockchain: Currency and Finance

Blockchain Ledger



- Ledger of transactions

Blockchain Ledger



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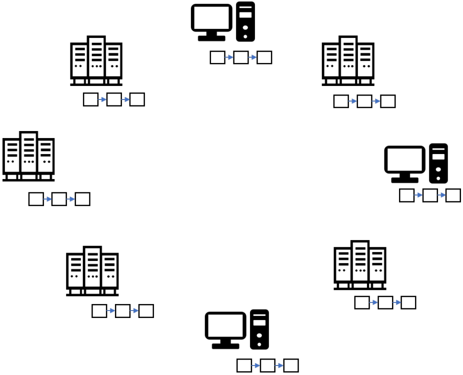

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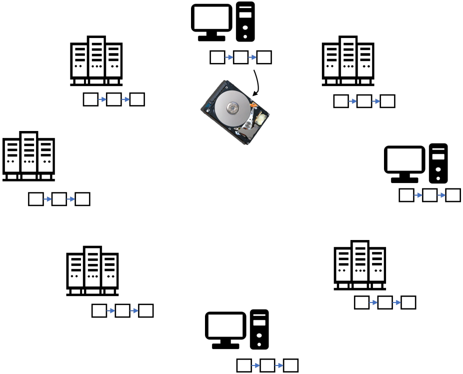

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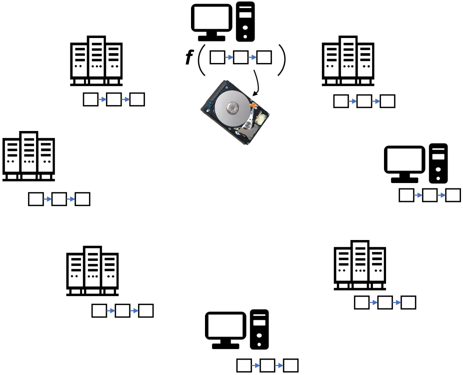

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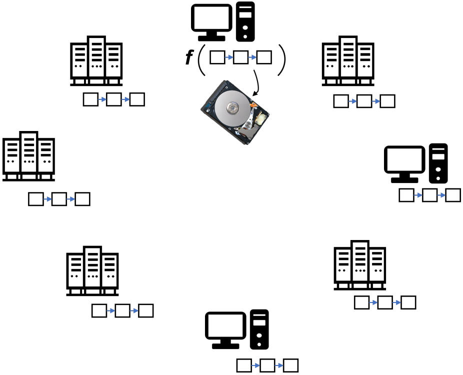

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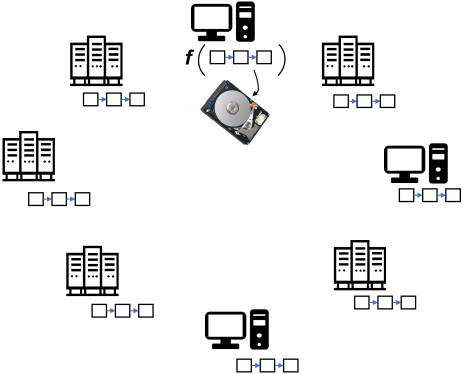
NVM technologies like persistent memories: low latency, high reliability

Blockchain Ledger



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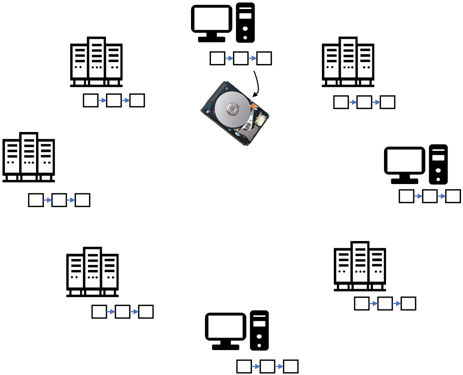
NVM technologies like persistent memories: low latency, high reliability
↳ Faster blockchains

Central Problem: Prohibitive Storage Overhead



- Each node stores a copy of the ledger in its memory

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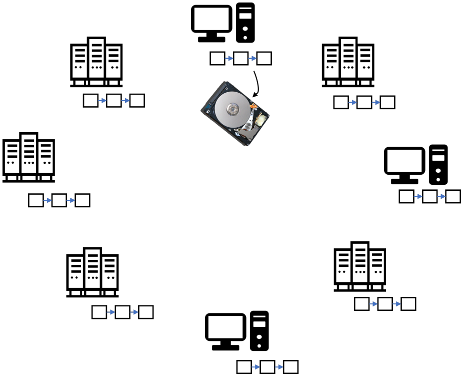

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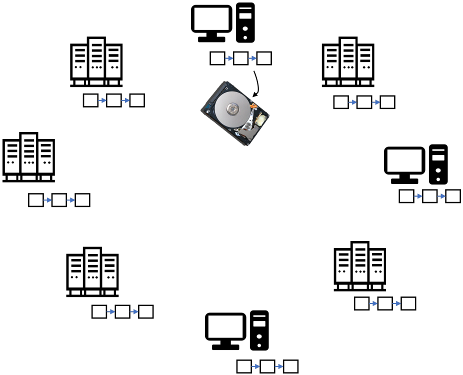

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Significant storage overhead

- Bitcoin ledger size $\sim$ 300GB[1]
- Ethereum ledger size $\sim$ 650GB[2]

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[2]https://etherscan.io/chartsync/chaindefault

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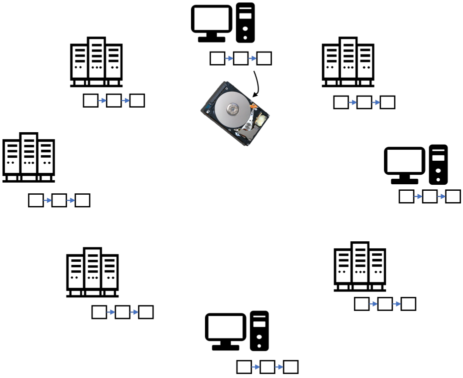

- Each node stores a copy of the ledger in its memory
- Prohibitive for resource limited nodes

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Allowing Light Nodes

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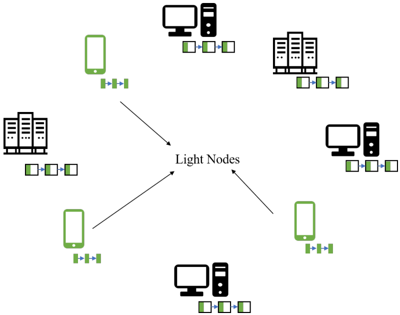

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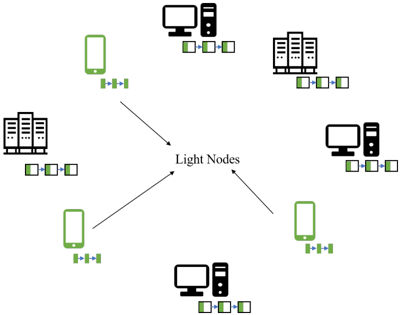

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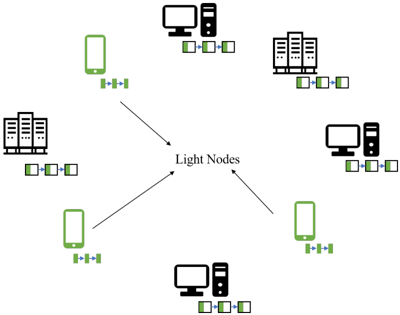

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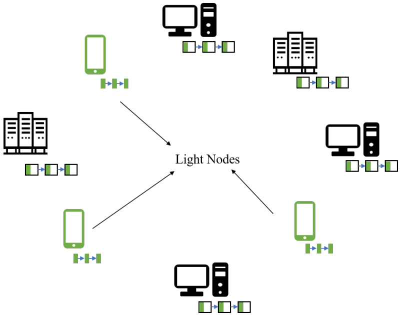

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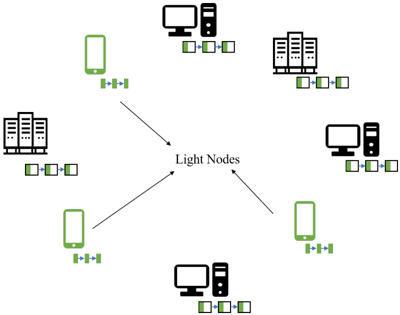

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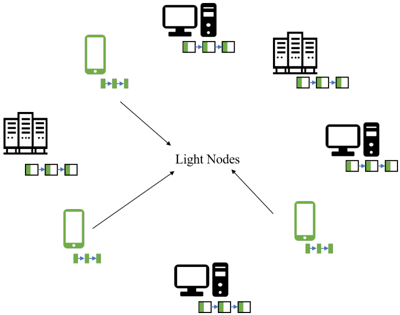

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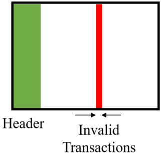
- Full nodes send verifiable fraud proofs to the light nodes to reject invalid blocks

Data Availability(DA) Attack

Systems with light nodes and a dishonest majority of full nodes are vulnerable to DA attacks [Al-Bassam '18], [Yu '19]

Adversary creates an invalid block



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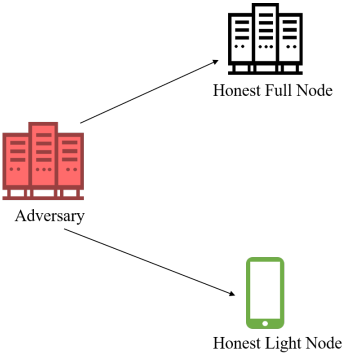

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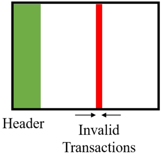

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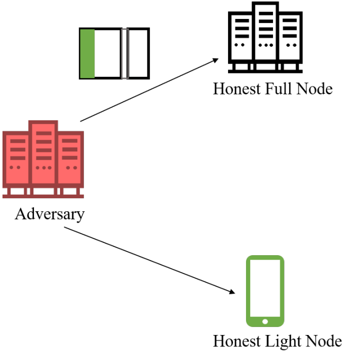

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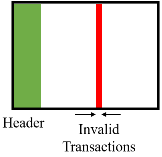

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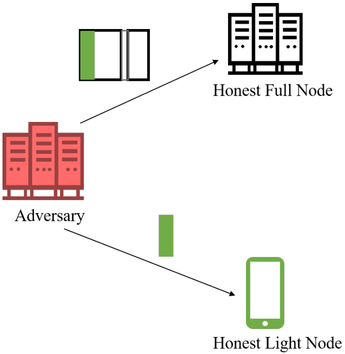

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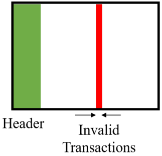

- Adversary: Provides block to Full node but hides invalid portion
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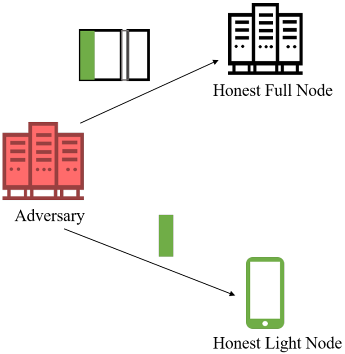

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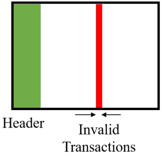

- Adversary: Provides block to Full node but hides invalid portion
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- Honest Nodes: Cannot verify missing transactions

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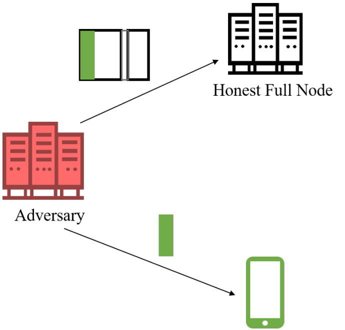

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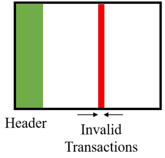

- Adversary: Provides block to Full node but hides invalid portion
  Provides header to Light node
- Honest Nodes: Cannot verify missing transactions → No fraud proof

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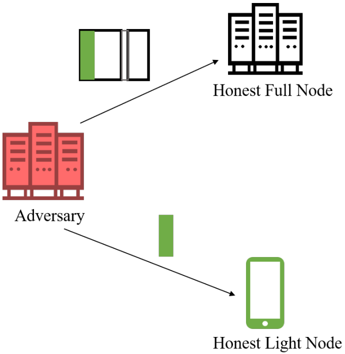

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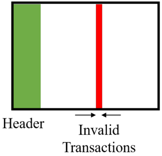

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- Light Nodes: No fraud proof

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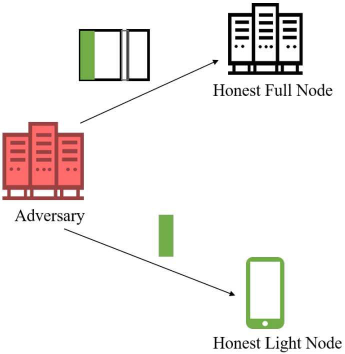

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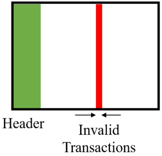

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- Honest Nodes: Cannot verify missing transactions $\rightarrow$ No fraud proof
- Light Nodes: No fraud proof $\rightarrow$ Accept the header

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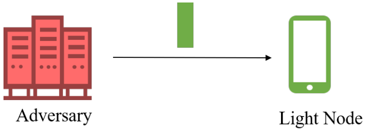

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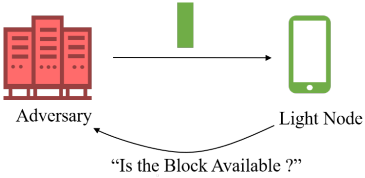

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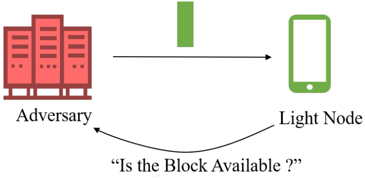

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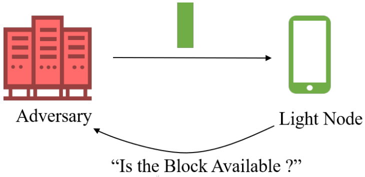

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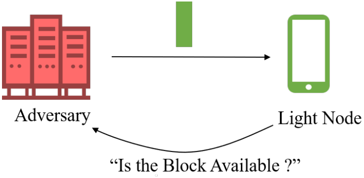

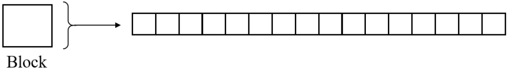

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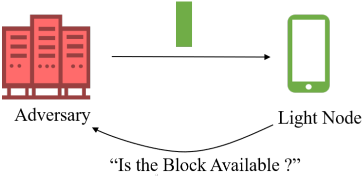

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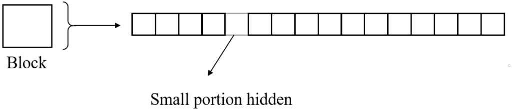

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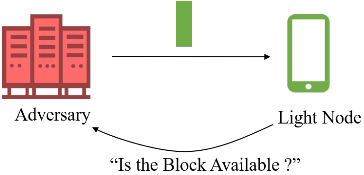

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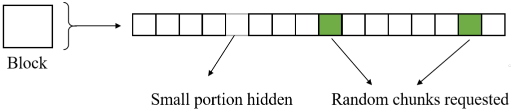

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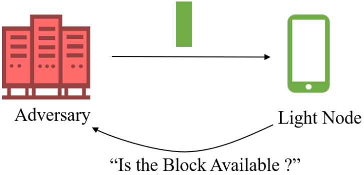

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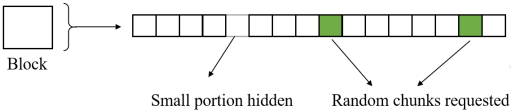
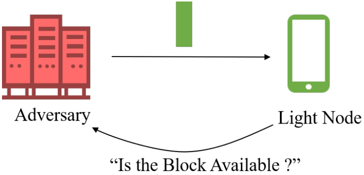


Probability of failure using 2 random samples:

Ensuring Data Availability



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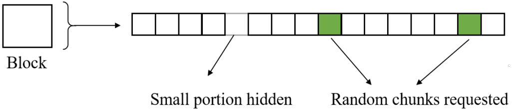

Probability of failure using 2 random samples:

$$\left(1-\frac{1}{16}\right)\left(1-\frac{1}{15}\right)=0.87$$

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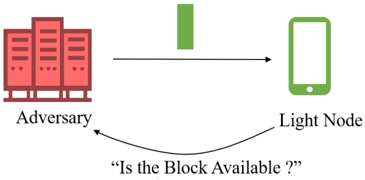

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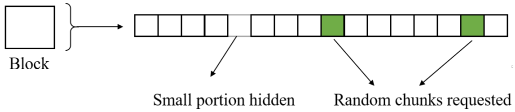
No coding:



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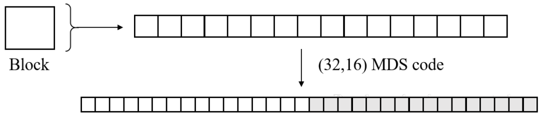
Erasure coding:



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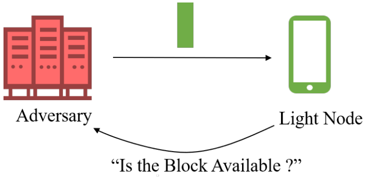

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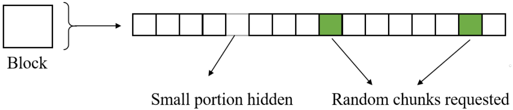

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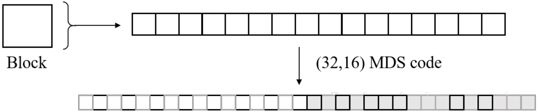
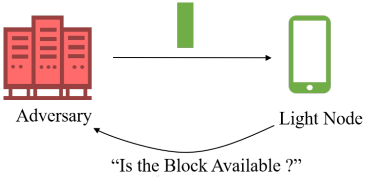
$$\left(1-\frac{1}{16}\right)\left(1-\frac{1}{15}\right)=0.87$$

Erasure coding:

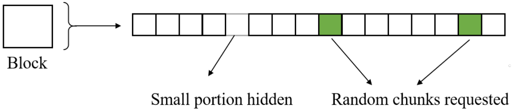


Ensuring Data Availability



- Request/sample few random chunks of the block
- Adversary can hide a small portion

No coding:



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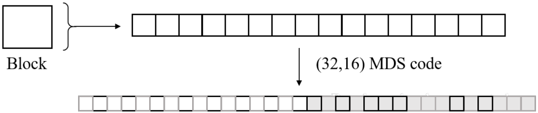

Probability of failure using 2 random samples:

$$\left(1-\frac{17}{32}\right)\left(1-\frac{17}{31}\right) = 0.21$$

Choice of Code Matters

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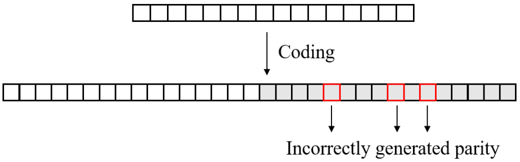
- Incorrect coding attack:

Choice of Code Matters



- Incorrect coding attack:
  - Adversary sends incorrectly coded block to Full Nodes

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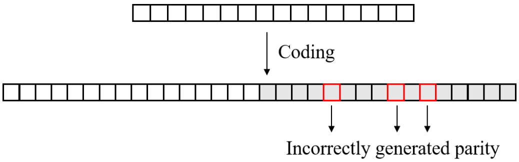

- Incorrect coding attack:
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  - Incorrect coding proof size: $\mathcal{O}$(sparsity of parity check equations)

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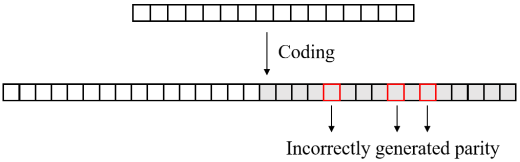

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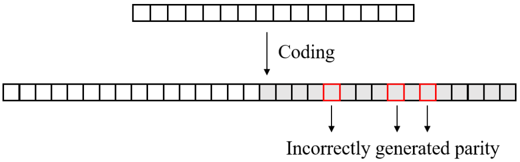

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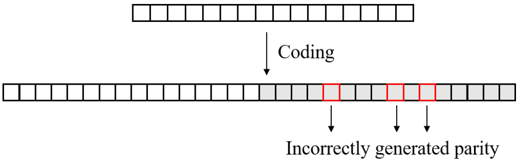

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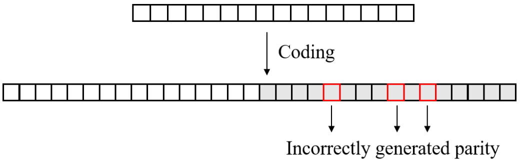

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  - Probability of Light node failure using s random samples $= (1-\alpha)^s$

LDPC Codes: A Strong Contender

LDPC codes:

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circles: variable nodes (VNs)
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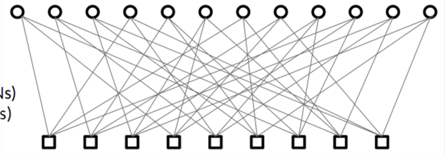

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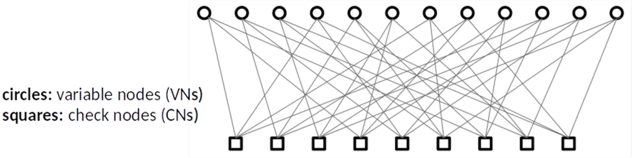

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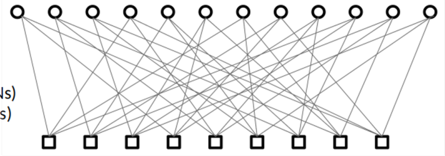

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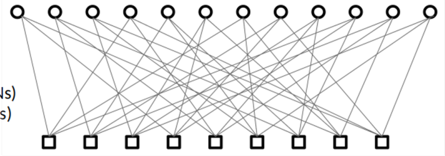

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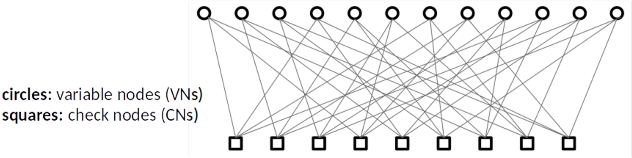

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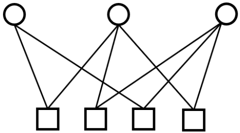
- Small incorrect coding proof size due to small check node degree
- Linear decoding in terms of the block size using peeling decoder
- What about the undecodable ratio?

Challenge with LDPC Codes: Small Stopping Sets

- Substructure in the Tanner Graph



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- Substructure in the Tanner Graph
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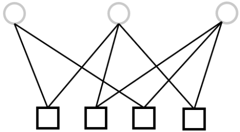

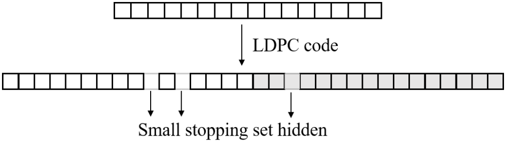

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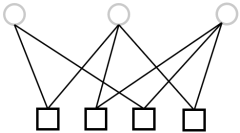

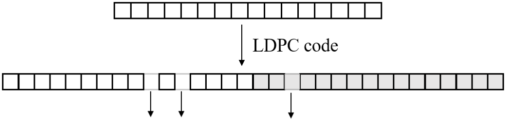

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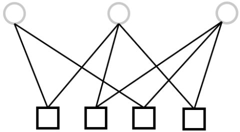

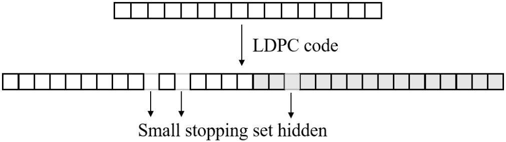

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Our work: Design of specialized LDPC codes with a coupled sampling strategy to achieve a significantly lower probability of failure.

Motivation: Not all VNs are equal

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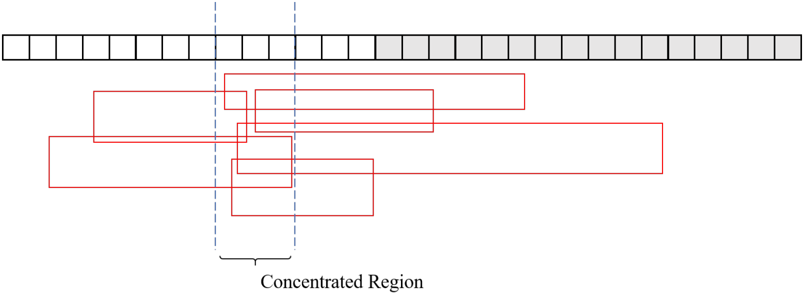
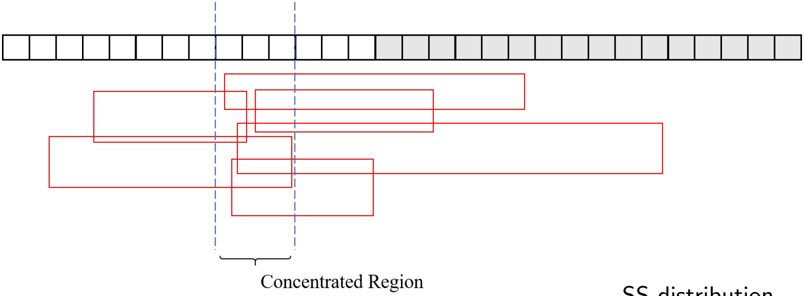
Concentrated Stopping Set Design



Code Design Idea:

- Concentrate stopping sets to a small section of VNs

Concentrated Stopping Set Design



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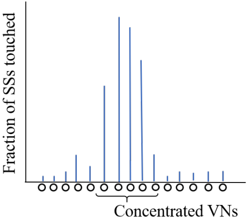

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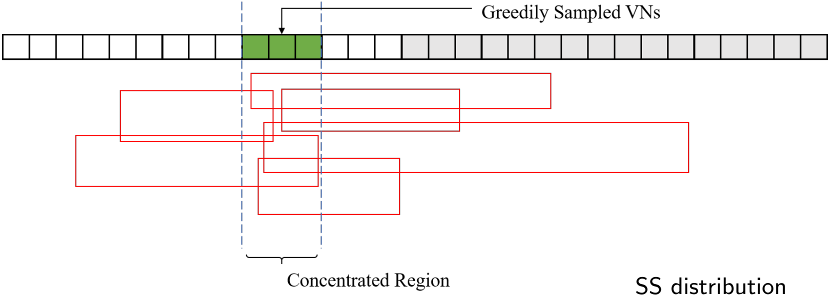
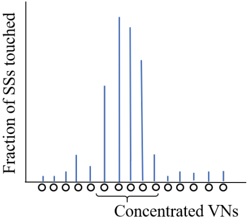


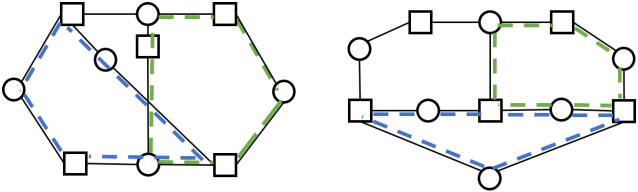
Code Design Idea:

- Concentrate stopping sets to a small section of VNs
- Greedily Sample this small section of VNs

SS distribution



How to Concentrate Stopping Sets?



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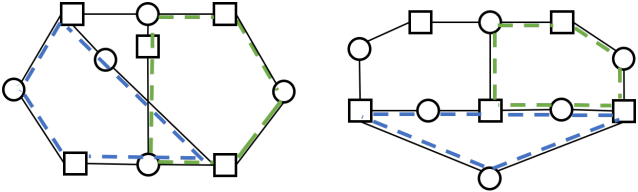

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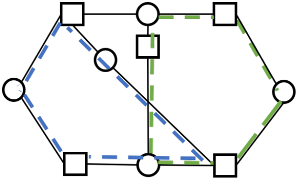

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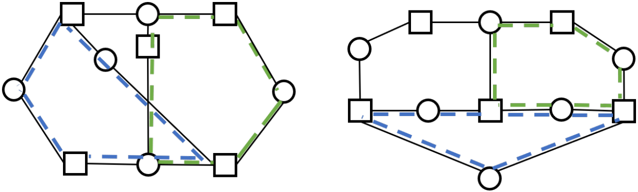

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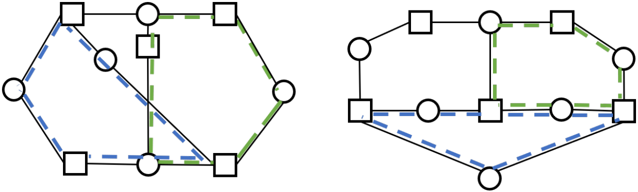

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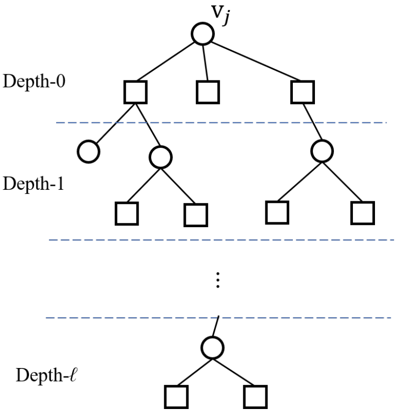
- How to design codes with concentrated cycles?
  We do so by modifying the well-known Progressive Edge Growth (PEG) algorithm

PEG Algorithm

- Constructs a Tanner Graph in an edge by edge manner [Xiao '05]

PEG Algorithm



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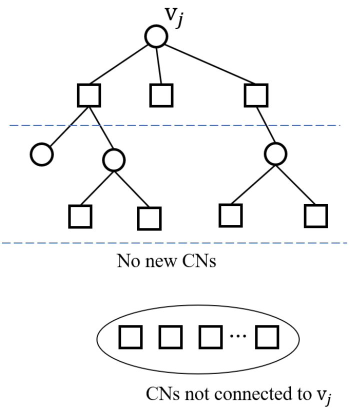
For each VN v_j
  Expand Tanner Graph in a BFS fashion

PEG Algorithm



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For each VN v_j
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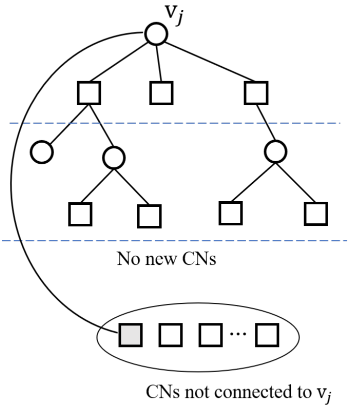
PEG Algorithm



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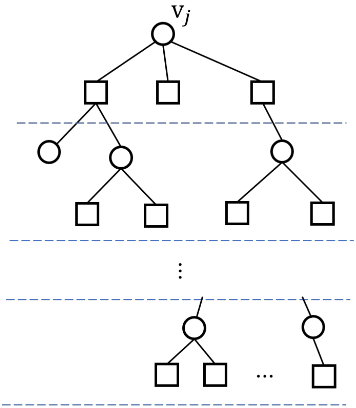
PEG Algorithm



All CNs exhausted

- Constructs a Tanner Graph in an edge by edge manner [Xiao '05]

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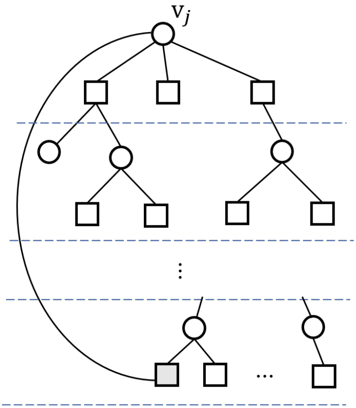

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Else
- Find CNs most distant to v_j
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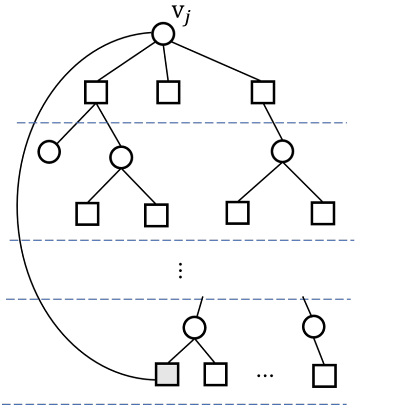
PEG Algorithm



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New cycles created

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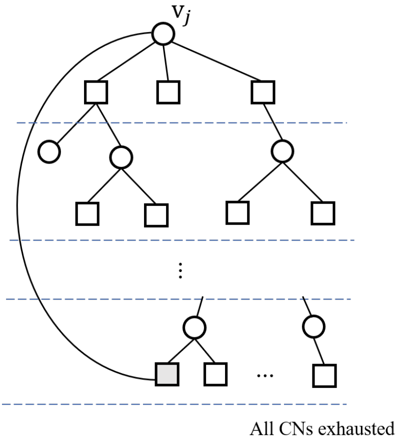

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New cycles created

We modify the CN selection criteria in green to concentrate cycles

Using Entropy to Concentrate Cycles

For distribution $p = (p_1, p_2, \ldots, p_n)$, Entropy $\mathcal{H}(p) = \sum_{i=1}^{n} p_i \log \frac{1}{p_i}$

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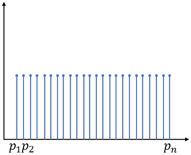

High Entropy

Using Entropy to Concentrate Cycles

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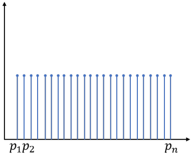
- Uniform distributions have high entropy
- Concentrated distributions have low entropy



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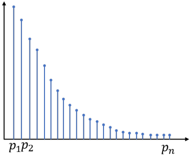
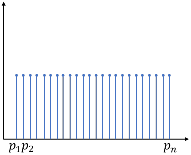


Low Entropy

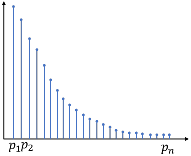
Using Entropy to Concentrate Cycles

For distribution $p = (p_1, p_2, \ldots, p_n)$, Entropy $\mathcal{H}(p) = \sum_{i=1}^{n} p_i \log \frac{1}{p_i}$

- Uniform distributions have high entropy
- Concentrated distributions have low entropy



High Entropy



Low Entropy

We want the cycle distributions to be concentrated

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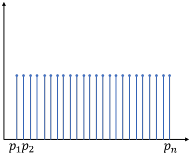

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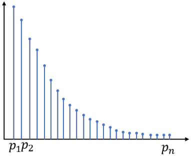

Low Entropy

We want the cycle distributions to be concentrated
$\rightarrow$ Select CNs such that the entropy of the cycle distribution is minimized

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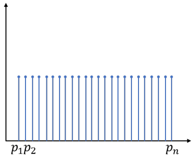
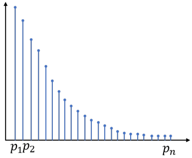


High Entropy



Low Entropy

EC (Entropy Constrained)-PEG Algorithm

For each VN v_j

Expand Tanner Graph in a BFS fashion

If $\exists$ CNs not connected to v_j

- select a CN with min degree not connected to v_j

Else *New cycles created*

We want the cycle distributions to be concentrated

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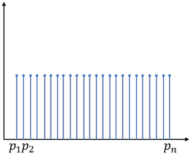

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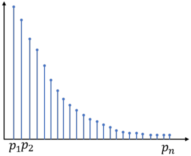

Low Entropy

EC (Entropy Constrained)-PEG Algorithm

For each VN v_j

Expand Tanner Graph in a BFS fashion

If $\exists$ CNs not connected to v_j

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- Find CNs most distant to v_j

We want the cycle distributions to be concentrated

$\rightarrow$ Select CNs such that the entropy of the cycle distribution is minimized

Using Entropy to Concentrate Cycles

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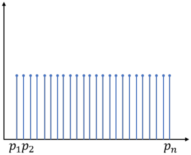

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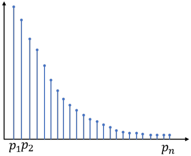

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EC (Entropy Constrained)-PEG Algorithm
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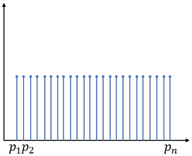

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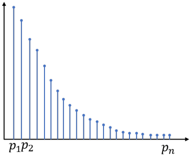

Low Entropy

EC (Entropy Constrained)-PEG Algorithm

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Expand Tanner Graph in a BFS fashion

If $\exists$ CNs not connected to v_j

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Else *New cycles created*

- Find CNs most distant to v_j
- Select CN that results in minimum entropy of resultant cycle distribution
- Update cycle distribution

We want the cycle distributions to be concentrated

$\rightarrow$ Select CNs such that the entropy of the cycle distribution is minimized

EC-PEG Algorithm

- Whenever a new edge, that creates cycles, is added to the Tanner Graph, we update the cycle counts of each VN

EC-PEG Algorithm

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VNs $(v_1, v_2, \ldots, v_n)$

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- $\lambda_i^{(g)} :=$ No. of cycles of length g that v_i is a part of, $g = 4, 6, 8$

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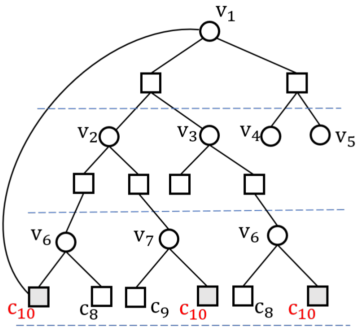

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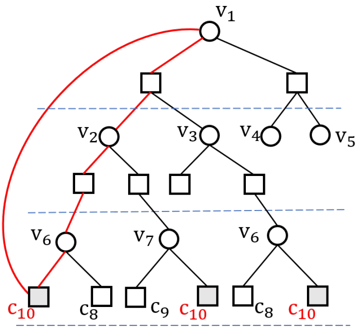

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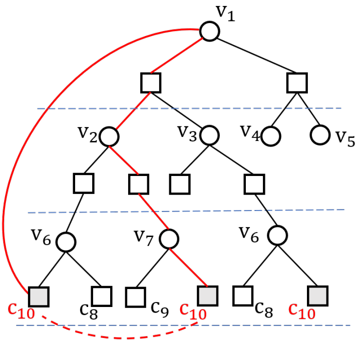

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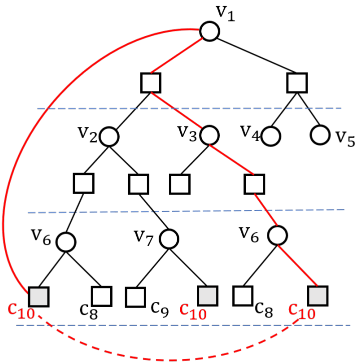

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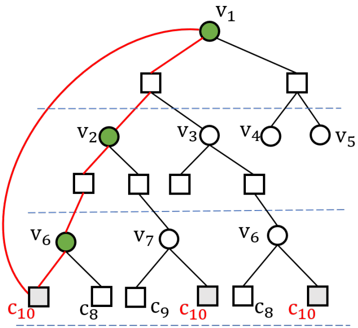

VNs $(v_1, v_2, \ldots, v_n)$

- $\lambda_i^{(g)}$:= No. of cycles of length g that v_i is a part of, $g = 4, 6, 8$
- $$\lambda_1^{(6)} = \lambda_1^{(6)} + 1$$ $$\lambda_2^{(6)} = \lambda_2^{(6)} + 1$$ $$\lambda_6^{(6)} = \lambda_6^{(6)} + 1$$

EC-PEG Algorithm

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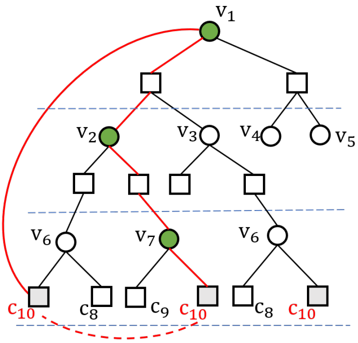

VNs $(v_1, v_2, \ldots, v_n)$

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  $$\lambda_7^{(6)} = \lambda_7^{(6)} + 1$$

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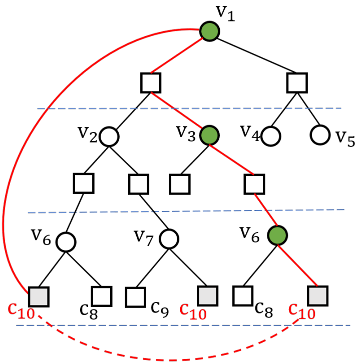

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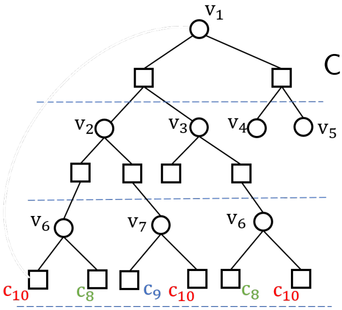
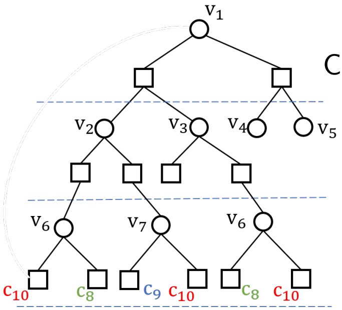
  $$\lambda_6^{(6)} = \lambda_6^{(6)} + 1$$

EC-PEG Algorithm: CN Selection Procedure



Candidate CNs : c_8, c_9, c_{10}

EC-PEG Algorithm: CN Selection Procedure



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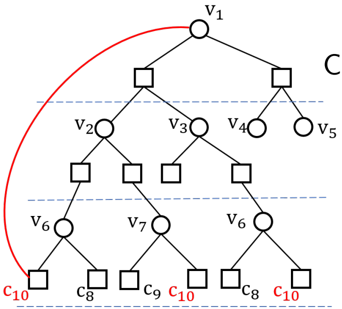
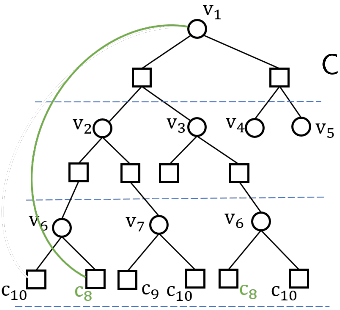


Candidate CNs : c_8, c_9, c_{10}

- For each CN candidate, calculate the resultant VN cycle counts
- $(\lambda_1^{(4)}, \dots, \lambda_n^{(4)}), (\lambda_1^{(6)}, \dots, \lambda_n^{(6)}), (\lambda_1^{(8)}, \dots, \lambda_n^{(8)})$

EC-PEG Algorithm: CN Selection Procedure



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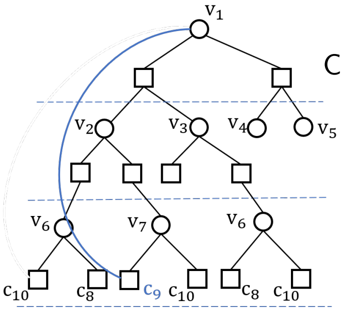

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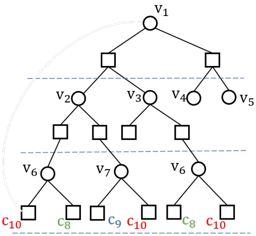

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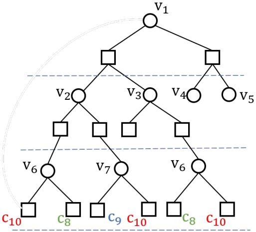

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$$\underbrace{(\lambda_1^{(g)}, \ldots, \lambda_n^{(g)})}_{\text{cycle counts}}$$

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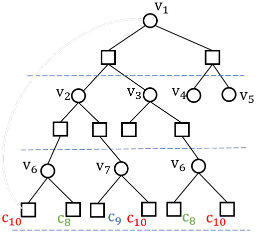

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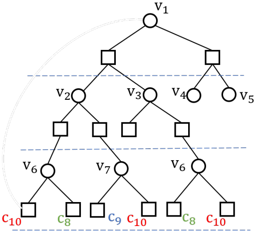
EC-PEG Algorithm: CN Selection Procedure



- $(\lambda_1^{(4)}, \ldots, \lambda_n^{(4)}), (\lambda_1^{(6)}, \ldots, \lambda_n^{(6)}), (\lambda_1^{(8)}, \ldots, \lambda_n^{(8)})$
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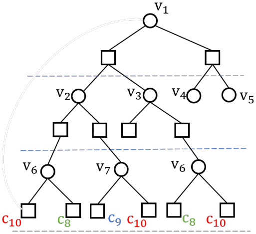

- $(\lambda_1^{(4)},\dots,\lambda_n^{(4)}),(\lambda_1^{(6)},\dots,\lambda_n^{(6)}),(\lambda_1^{(8)},\dots,\lambda_n^{(8)}) \rightarrow \mathcal{H}(\frac{\alpha^{(4)}+\alpha^{(6)}+\alpha^{(8)}}{3})$
- $(\lambda_1^{(4)},\dots,\lambda_n^{(4)}),(\lambda_1^{(6)},\dots,\lambda_n^{(6)}),(\lambda_1^{(8)},\dots,\lambda_n^{(8)})$
- $(\lambda_1^{(4)},\dots,\lambda_n^{(4)}),(\lambda_1^{(6)},\dots,\lambda_n^{(6)}),(\lambda_1^{(8)},\dots,\lambda_n^{(8)})$

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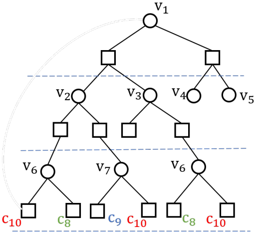

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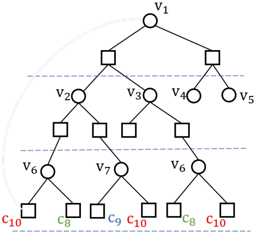

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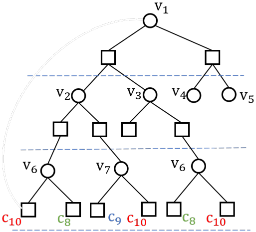

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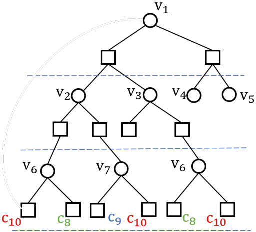
CN selection procedure:

EC-PEG Algorithm: CN Selection Procedure



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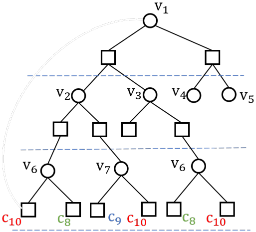

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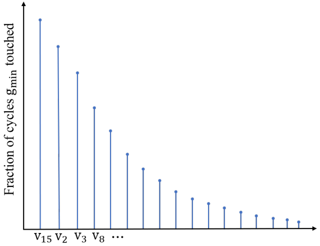
Note:

- Minimizing the entropy of joint cycle counts ensures that all cycle distributions are concentrated towards the same set of VNs

Sampling Strategy

- Our sampling strategy greedily samples VNs that are part of a large number of cycles



$g =$ smallest cycle length in Tanner Graph $\mathcal{G}$
While sample set size $< s$
- $v =$ VN that is part of largest no. of cycles of length g in $\mathcal{G}$
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- remove v and all incident edges from $\mathcal{G}$

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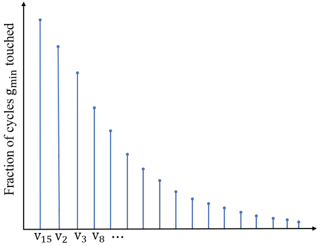

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If $\nexists$ cycles of length g in $\mathcal{G}$
- $g = g + 2$

Simulation Results

- Code parameters: Code length $= 100$, VN degree $= 4$, Rate $= \frac{1}{2}$, girth $= 6$.

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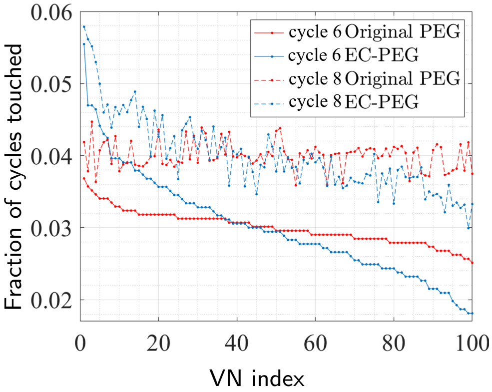

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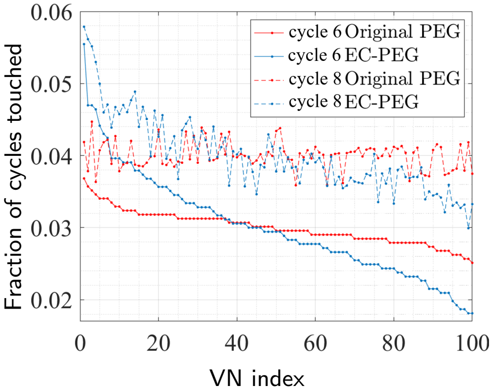

- VN indices arranged in decreasing order of cycle 6 fractions
- Cycle 6 and cycle 8 concentrated towards same set of VNs

Simulation Results

Fraction of SSs of size 11, 12 touched by different VNs

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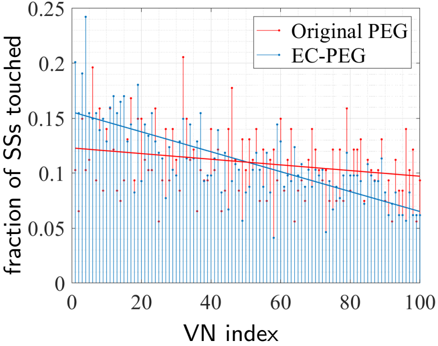
SSs of size 11



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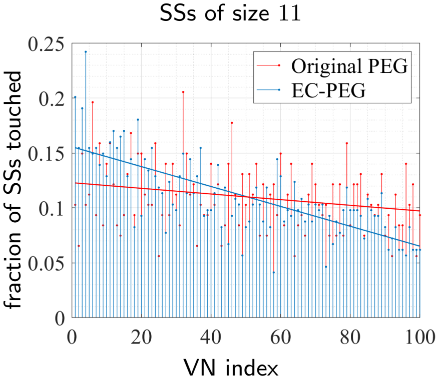

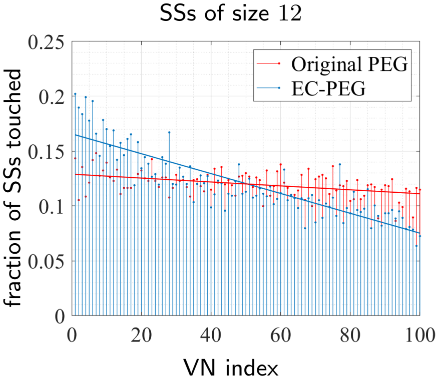

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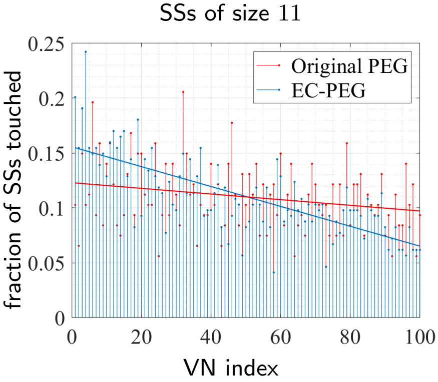

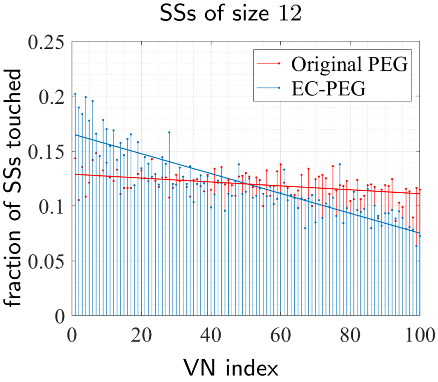

- VN indices arranged in decreasing order of cycle 6 fractions
- SSs are concentrated towards the same set of VNs as the cycles

Simulation Results

Probability of failure for a stopping set of size μ

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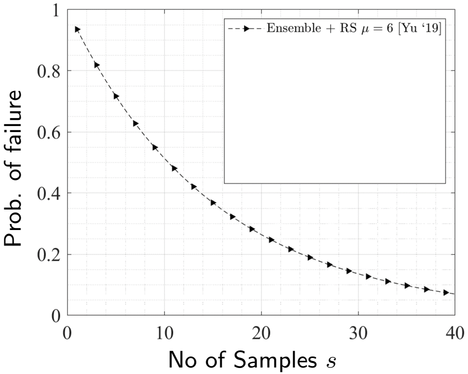
RS: Random Sampling



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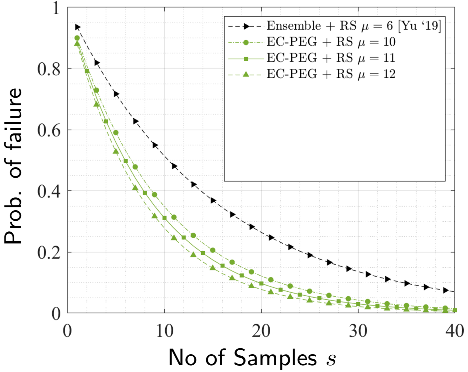

Simulation Results

Probability of failure for a stopping set of size μ

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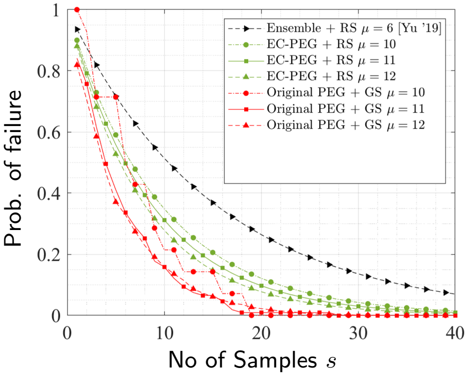

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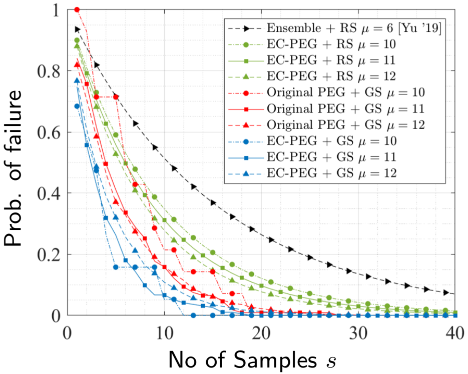

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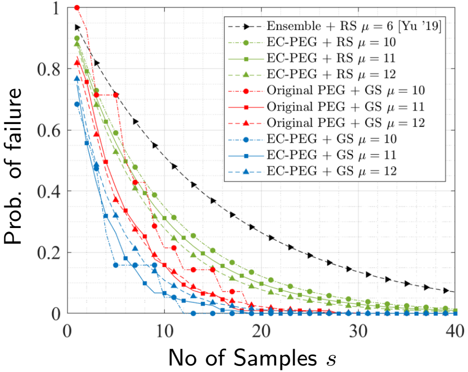

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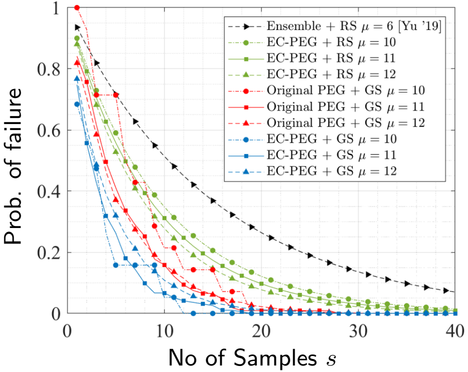

- Concentrated LDPC codes with Greedy sampling improve the probability of failure
  ↳ Note that the probability of failure depends on the fraction of stopping sets touched (by greedy sampling) and not the actual number.

Incorrect Coding Proof Size

- Depends on the maximum check node degree

Rate	Code length	VN degree	Ensemble [Yu '19]	PEG	EC-PEG
$\frac{1}{2}$	100	4	16	9	11
	200	4	16	9	15
$\frac{1}{4}$	100	4	8	7	10
	200	4	8	6	9

Table: Maximum CN degree for different codes.

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- Concentrated LDPC codes do not sacrifice on the incorrect coding proof size

Conclusion and Extentions

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- Extensions (Mitra '21):
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  - Provided optimal sampling strategies and associated coupled LDPC code construction to improve the security against such strong adversaries for a given sample complexity

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