Coding on Barrier Channels beyond Guaranteed Correction

Yuval Ben-Hur and Yuval Cassuto

The Andrew and Erna Viterbi Department of Electrical and Computer Engineering
Technion - Israel Institute of Technology

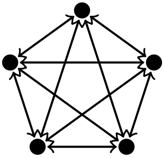
NVMW 2022 – San Diego, California, USA

Outline

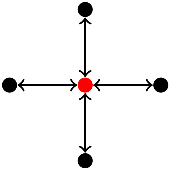
1. Motivation
2. Dual-parameter Barrier channel: definition and capacity
3. Prior work: code construction
4. Decoding beyond guaranteed correction
    - Simplified ML decoding
    - Enhanced-performance practical decoders
5. Conclusion

Introduction

- Efficient information systems usually span multiple representation levels per channel use.
- The most common multi-bit scaling model is the Q-ary symmetric channel.
- We study the **Barrier channel** – a unique scaling principle where all errors are either to or from a specific level/symbol.



Symmetric

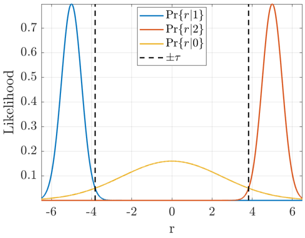


Barrier

Motivation (1)

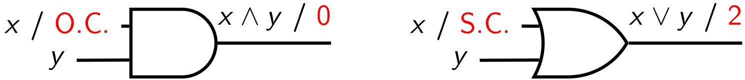
- **Binary** systems that have an **additional representation level**.
  - Increased representation power ($\sim 60\%$).
  - Non-symmetric error model.

- **Some possible applications**: NextGen EEPROM [Bitouze, 2010], Wideband COMM [Vladimirov, 2017], IoT biosensors [Tang, 2020].
- **Ternary-Input AWGN channel**: Transitions between extreme levels are negligible.

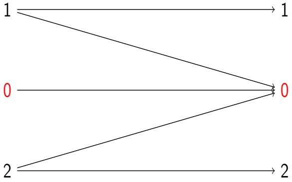


Motivation (2)

- **In-memory processing** [Luo, 2020]: Enhanced efficiency of memristive in-memory logic using **Ternary logic**.
- **Switching faults** may cause barrier errors, e.g.:
  - Open-circuit (O.C.) fault in AND gate outputs 0.
  - Short-circuit (S.C.) fault in OR gate outputs 2.



- **Model**: Barrier channel applied on computation result.

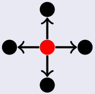


Channel definition

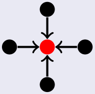
Definition: Q-ary Dual-Parameter Barrier channel

Let $Q \in \mathbb{N}$ and let $0 \leq p, q \leq 1$. $W_Q(p, q)$ is defined as follows

$$P(Y|X = 0) = \begin{cases} 1 - q, & Y = X \\ q/(Q-1), & Y \neq X \end{cases},$$



$$P(Y|X \neq 0) = \begin{cases} 1 - p, & Y = X, \\ p, & Y = 0, \\ 0, & \textit{otherwise} \end{cases}$$



Several specific versions of $W_Q(p, q)$ were already studied.

- $W_2(p, q)$ is the *binary asymmetric channel* $BAC(p, q)$.
- $W_Q(p, 0)$ is the *Q-ary Asymmetric Parallel channel* [Ahlswede-Aydinian, 2008].
- $W_3(q/2, q)$ is the *Non-symmetric Ternary channel* [Bitouze et al., 2010].

Channel capacity of $W_3(p, q)$

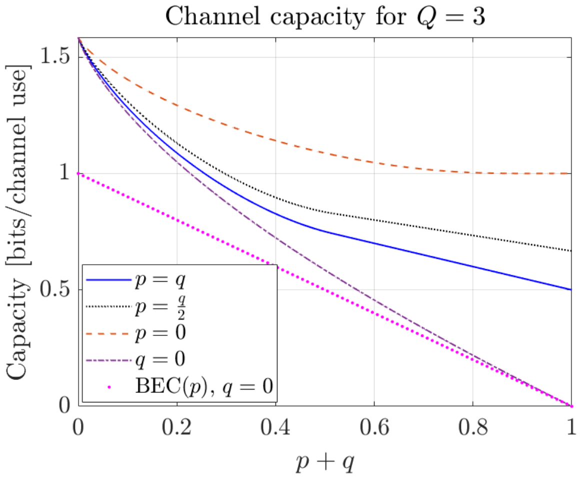
Theorem: Capacity of $W_3(p, q)$

Let $0 \leq p, q \leq 1$ such that $p + q < 1$. Define $\beta_{p,q}(\varphi) \triangleq q + (1 - p - q)\,\varphi$ and $\gamma(a) \triangleq h_2(a) + a$. Then,

$$\mathrm{Cap}\left(W_3(p, q)\right) = \gamma\left(\beta_{p,q}(\varphi^*)\right) - \varphi^* h_2(p) - (1 - \varphi^*)\gamma\left(q\right),$$

where $\varphi^* \triangleq \min\left[\frac{1-q-\left(1+2^{-\frac{\gamma(p)-1-h_2(q)}{1-p-q}}\right)^{-1}}{1-p-q}, 1\right].$

Channel capacity of $W_3(p, q)$



Capacity of $W_3(p, q)$ for several relations between p and q.

A code correcting t barrier errors

Code construction $\mathcal{C}$ [Bitouze et al., 2010]:

- θ from an $(n, \mathcal{M}_\Theta, 2t+1)_2$ code Θ.
- λ from an $(w_H(\theta), \mathcal{M}_\theta, t+1)_{Q-1}$ code Λ.

Construct Q-ary codeword $c \in \mathcal{C}$ using θ as an indicator for locations of non-zero symbols, whose values are given by λ.

Example: Codeword construction for $Q = 3$, $n = 8$, $t = 2$

θ	1	0	1	1	0	0	1	1	$\in \Theta[8, \mathcal{M}_\Theta, 5]_2$
λ	1	-	2	1	-	-	1	2	$\in \Lambda[5, \mathcal{M}_5, 3]_2$
c	1	0	2	1	0	0	1	2	$\in \mathcal{C}$

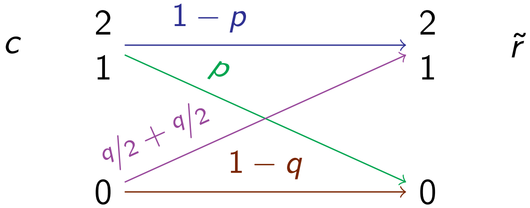
What about beyond worst-case correction?

Coordinate characterization for dual-parameter (p,q) channel

Definition: Coordinate-set partition

$\bigcup_{s=0}^{4} \mathcal{S}_s = \{1, \ldots, n\}$ where

Coordinate set	Channel transition	Log-Likelihood (ℓ_s)
$\mathcal{S}_0$	$\tilde{r}_i = c_i = 0$	$\log(1-q)$
$\mathcal{S}_1$	$\tilde{r}_i = c_i \neq 0$	$\log(1-p)$
$\mathcal{S}_2$	$\tilde{r}_i = 0, c_i \neq 0$	$\log(p)$
$\mathcal{S}_3$	$\tilde{r}_i \neq 0, c_i = 0$	$\log(q/(Q-1))$
$\mathcal{S}_4$	$\tilde{r}_i \neq 0, c_i \neq 0, \tilde{r}_i \neq c_i$	$-\infty$



Maximum-Likelihood decoding

Theorem: MLD for $W_Q(p, q)$

The ML codeword $c^{ML} \in \mathcal{C}$ is the one that

$$\text{maximizes } \sum_{s=0}^{3} \ell_s |\mathcal{S}_s|, \text{ subject to } |\mathcal{S}_4| = 0.$$

Constituent binary MLDs - to be used in new decoding algorithm:

Component 1: Indicator MLD

Input: $\tilde{r}$

Find $\theta \in \Theta$ that maximizes $\sum_{s=0}^{3} \ell_s |\mathcal{S}_s| + \ell_1 |\mathcal{S}_4|$.

Output: θ^{ML}

Component 2: Residual MLD

Input: $\tilde{r}$, θ

Find $\lambda \in \Lambda_{w_H(\theta)}$ satisfying $\lambda_i = \tilde{r}_i$ for every i such that $\theta_i = 1, \tilde{r}_i \neq 0$.

Output: λ if unique, failure otherwise.

Decomposed MLD for $\mathcal{C}$

Algorithm 1

Input : $\tilde{r}$ - channel output
Output : $\hat{c}$ - decoded codeword (or "decoding failure")
Initialize: $\Theta' \leftarrow \Theta$
while not returned **do**
  $\hat{\boldsymbol{\theta}} \leftarrow$ indicator MLD with input $\tilde{r}$ and code Θ'
  $\hat{\boldsymbol{\lambda}} \leftarrow$ residual MLD with input $\tilde{r}$ and code $\Lambda_{w_H(\hat{\boldsymbol{\theta}})}$
  if "no solution" **then**
    $\Theta' \leftarrow \Theta' \setminus \hat{\boldsymbol{\theta}}$
  else if "multiple solutions" **then**
    return *decoding failure*
  else
    reconstruct $\hat{c}$ from $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\lambda}}$ and return
  end if
end while

From MLD to CLD

- Algorithm 1 is equivalent to ML decoding for $\mathcal{C}$.

Theorem:

Algorithm 1 (Decomposed MLD) outputs c^{ML}, if unique.

- However, it is still impractical to implement a decoder that iterates over the entire code Θ.
- Inspired by the sequential nature of decomposed MLD, we propose ***Cooperative List Decoding*** (CLD).

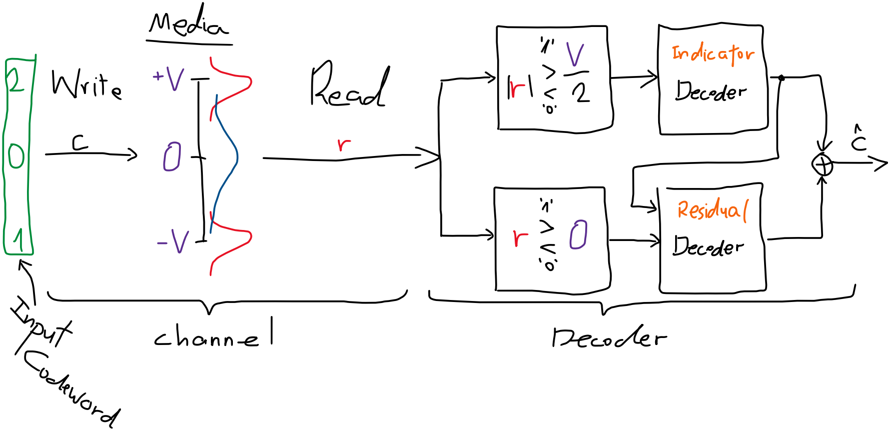
CLD for $\mathcal{C}$

Input : $\tilde{r}$ - channel output, L - list-size
Output : $\hat{c}$ - decoded codeword (or "decoding failure")

Initialize: $\Theta' \leftarrow \{\theta_l\}_{l=1}^{L} \leftarrow$ list-decoder for Θ

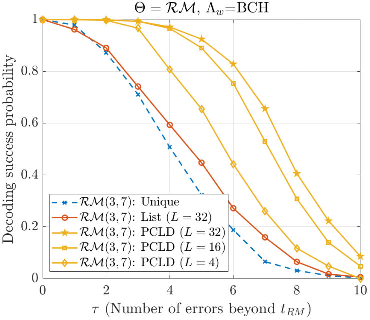
while not returned **do**
  $\hat{\boldsymbol{\theta}} \leftarrow$ indicator MLD with input $\tilde{r}$ and code Θ'
  $\hat{\boldsymbol{\lambda}} \leftarrow$ residual MLD with input $\tilde{r}$ and code $\Lambda_{w_H(\hat{\boldsymbol{\theta}})}$
  if "no solution" **then**
    $\Theta' \leftarrow \Theta' \setminus \hat{\boldsymbol{\theta}}$
  else if "multiple solutions" **then**
    return *decoding failure*
  else
    reconstruct $\hat{c}$ from $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\lambda}}$ and return
  end if
end while

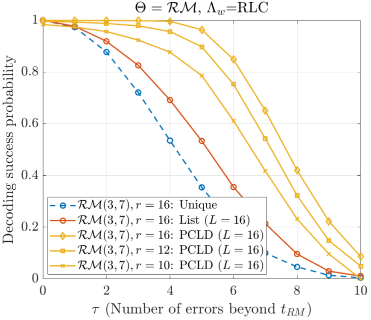
Conceptual simulation flow



Simulations for performance evaluation

- Indicator code Θ is a Reed-Muller code $\mathcal{RM}(3,7)$ with $t_{RM} = 7$.
- Residual code Λ is shortened BCH or linear random-code (RLC).





Conclusion

Dual-parameter Barrier channel $W_Q(p,q)$ - an interesting error model.

Our main contributions include:

- Capacity for $Q = 3$.
- Simplified ML decoder.
- Practical decoders.

Future work:

- Capacity for general Q.
- Codes for specific (p,q).
- Decoding without a list-decoder.