Fast Nested Decoding for Short Generalized Integrated Interleaved BCH Codes

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***Abstract*—Generalized integrated interleaved (GII) codes that nest BCH sub-codewords to form more powerful BCH codewords are among the best error-correcting codes for storage class memories (SCMs). However, SCMs require short codeword length and low redundancy. In this case, miscorrections on the sub-words lead to severe error-correcting performance degradation if untreated. This abstract presents recent results on low-complexity miscorrection mitigation schemes for GII codes as well as their optimizations for latency reduction. For an example GII-BCH code with 3-error-correcting sub-codewords, the optimizations reduce the average nested decoding latency by 43% at input bit error rate 10^{-3} with very small performance loss.**

I. Introduction

Generalized integrated interleaved (GII) codes can nest BCH sub-codewords to form more powerful BCH codewords [1], [2]. GII decoding consists of two stages: sub-word decoding and nested decoding. In most cases, only the first-stage decoding of individual BCH sub-words is needed. Hence, very high throughput can be achieved with low complexity. Besides, extra errors are correctable by the nested decoding. Due to their hyper-speed decoding and excellent correction capability, GII-BCH codes are among the best codes for storage class memories (SCMs).

SCMs require short codeword length and low redundancy, such as 2560 data bits protected by 256 parity bits. In this case, the BCH sub-codewords have small correction capability, such as t_0=3. A received word may be decoded as another codeword when the number of errors is beyond the correction capability. Such miscorrection has been well-studied for traditional BCH codes. However, the miscorrections for GII codes have not been previously investigated.

This abstract presents our results [3], [4] on low-latency and low-complexity miscorrection mitigation schemes for short GII-BCH code decoding. For the first time, our research [3] shows that miscorrections degrade GII decoding performance by orders of magnitudes. The GII decoding schemes in [5] and [6] only handle a limited number of error patterns that lead to miscorrections and do not close the performance gap.

Three low-complexity miscorrection mitigation methods were proposed in [3] to bring the actual frame error rate (FER) of GII decoding very close to the theoretical FER. However, the higher-order nested syndrome computation before each nested decoding round leads to significant decoding latency overhead. Through analyzing the dominant error patterns leading to miscorrections, it was proposed in [4] to skip the nested syndrome checking when miscorrections are unlikely to happen. The resulted slight performance loss is further eliminated by exploiting 2-bit extended (e-) BCH codes and adding a few global parity bits [4]. For an example GII-BCH code with 3-error-correcting sub-codewords, the average nested decoding latency is reduced by 43% with negligible performance loss using these optimizations.

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II. GII-BCH Codes and Miscorrections

A GII-BCH $[m,v]$ code is constructed using $v+1$ BCH codes $\mathcal{C}_v \subseteq \mathcal{C}_{v-1} \subseteq \cdots \subseteq \mathcal{C}_1 \subset \mathcal{C}_0$ over $GF(2^q)$ with error-correction capabilities $t_v \geq t_{v-1} \geq \cdots \geq t_1 > t_0$ as [2]

$$\mathcal{C} \triangleq \Big\{[c_0(x), c_1(x), \cdots, c_{m-1}(x)] : c_i(x) \in \mathcal{C}_0, \\ \tilde{c}_l(x) = \sum_{i=0}^{m-1} \alpha^{il}(x) c_i(x) \in \mathcal{C}_{v-l}, 0 \leq l < v\Big\}, \quad (1)$$

where α is a primitive element of $GF(2^q)$ and $\alpha^{il}(x)$ is the standard basis polynomial representation of α^{il}. $c_i(x)$ and $\tilde{c}_l(x)$ are a sub-codeword and a nested codeword, respectively.

GII-BCH decoding has two stages. The first is the traditional t_0-error-correcting BCH decoding for each received sub-word $y_i(x)$. $2t_0$ syndromes are calculated as $S_j^{(i)} = y_i(\alpha^{j+1})$ $(0 \leq j < 2t_0)$. Then a key equation solver (KES) computes the error locator polynomial $\Lambda(x)$. The inverse roots of $\Lambda(x)$ are found by the Chien search and they are the error locations. Decoding failure is declared if the root number is not equal to $\deg \Lambda(x)$.

The second-stage nested decoding is activated when some sub-words fail the decoding or are miscorrected due to extra errors. $2t$ syndromes are required to correct t errors. Higher-order nested syndromes $\tilde{S}_j^{(l)}$ $(2t_0 \leq j < 2t_1)$ can be derived from $\tilde{y}_l(x) = \sum_{i=0}^{m-1} \alpha^{il}(x) y_i(x)$ since they are at least t_1-error-correcting. Let the indices of the $b \leq v$ sub-words with extra errors be $i_0, i_1, \cdots, i_{b-1}$. From (1), their higher-order syndromes can be calculated by

$$\left[S_j^{(i_0)}, S_j^{(i_1)}, \cdots, S_j^{(i_{b-1})}\right]^T = A^{-1}\left[\tilde{S}_j^{(0)}, \tilde{S}_j^{(1)}, \cdots, \tilde{S}_j^{(b-1)}\right]^T, \quad (2)$$

where the matrix entry $A_{u,w}$ equals $\alpha^{i_w u(j+1)}$ $(0 \leq u, w < b)$ [2]. Using the $2t_1$ syndromes, up to t_1 errors are correctable for each of the b sub-words. If some sub-words are still uncorrected, a similar process is repeated for up to v rounds.

For SCMs using around 256 parity bits to protect 2560 data bits, the GII-BCH [4,3] code over $GF(2^{10})$ with $[t_0, t_1, t_2, t_3] = [3, 5, 6, 11]$ can achieve a good tradeoff on FER

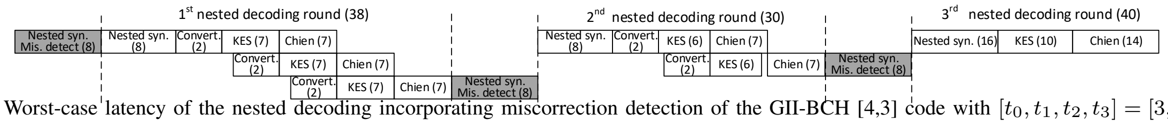

Fig. 1. Worst-case latency of the nested decoding incorporating miscorrection detection of the GII-BCH [4,3] code with $[t_0, t_1, t_2, t_3] = [3, 5, 6, 11]$.

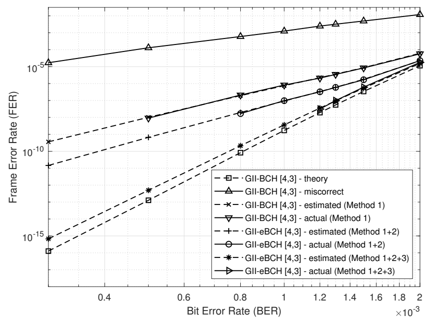

Fig. 2. FERs of GII-BCH [4,3] decoding over BSC [3].

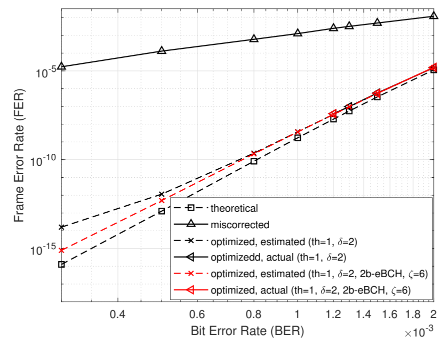

Fig. 3. FERs of GII-BCH [4,3] decoding with short-latency miscorrection mitigation schemes [4].

and decoding complexity [3]. The theoretical FER of this code over binary symmetric channel (BSC) and the FER taking into account miscorrections are plotted in Fig. 2. The miscorrections cause orders of magnitude degradation on the FER. The reason is that extra errors can not be corrected by the nested decoding process if the miscorrections are not identified in the sub-word decoding.

III. Miscorrection Mitigation Schemes and Optimizations

To bring the actual FER of GII decoding very close to its theoretical FER, three low-complexity miscorrection mitigation methods were proposed in [3]. The first is to test if any higher-order nested syndromes are nonzero. This can reliably detect miscorrections since the nested codewords are linear combinations of the sub-words and they are of higher correction capability. However, this method does not tell which sub-words are miscorrected. Only $v + 1 - i$ sub-words can be sent to the i-th nested decoding round. To better identify the miscorrected sub-words, the second method utilizes 1-bit eBCH codes. In this case, a sub-word is miscorrected if its bit-wise XOR result is different from the parity of the number of roots of its $\Lambda(x)$ polynomial found from the decoding. The third method keeps $\Lambda(x)$ with full length, if $deg(\Lambda(x))$ from the i-th decoding round is larger than t_i, then miscorrection happened. Combining these methods, the actual FER of GII decoding becomes very close to the theoretical FER. Formulas have also been developed in [3] to estimate the FERs of GII decoding with different miscorrection mitigation schemes and they match the simulation results at higher BER well.

The higher-order nested syndrome computation requires many clock cycles. Checking the nested syndromes for miscorrection detection at the end of each decoding round causes significant overhead to the overall nested decoding latency as shown in Fig. 1 for the example GII-BCH [4,3] code. In this figure, the numbers of clock cycles needed for the steps are written in the parentheses and they are derived from the settings for improving hardware efficiency.

Optimized miscorrection mitigation schemes were proposed in [4] to substantially reduce the nested decoding latency. In particular, the nested syndrome checking can be skipped when miscorrections are less likely to happen. It was discovered that the probability of a received sub-word being miscorrected decreases significantly with $\deg(\Lambda(x))$. Accordingly, the nested syndrome checking can be skipped if the $\deg(\Lambda(x))$ of each sub-word does not exceed a threshold th. Besides, completely skipping the nested syndrome checking after a later nested decoding round δ does not bring any noticeable difference in the FER. Fig. 3 shows that using $th = 1$ and $\delta = 2$ only lead to small FER degradation at lower BER. To make up this difference, it was also proposed in [4] to adopt 2-bit eBCH codes and add a few, such as $\xi = 6$, global parities that are the XOR results of segments of every sub-codeword. Formulas were also developed in [4] to estimate the FERs of these optimized miscorrection mitigation schemes and they match the simulation results well. Using these optimized schemes that have negligible hardware complexity overhead, the average nested decoding latency for the GII-BCH [4,3] code at BER 10^{-3} aggregated by the probability of activating each step is reduced from 8.3 to 4.7 clock cycles, which is 43% reduction.

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