Error-Correcting WOM Codes for Worst-Case and Random Errors

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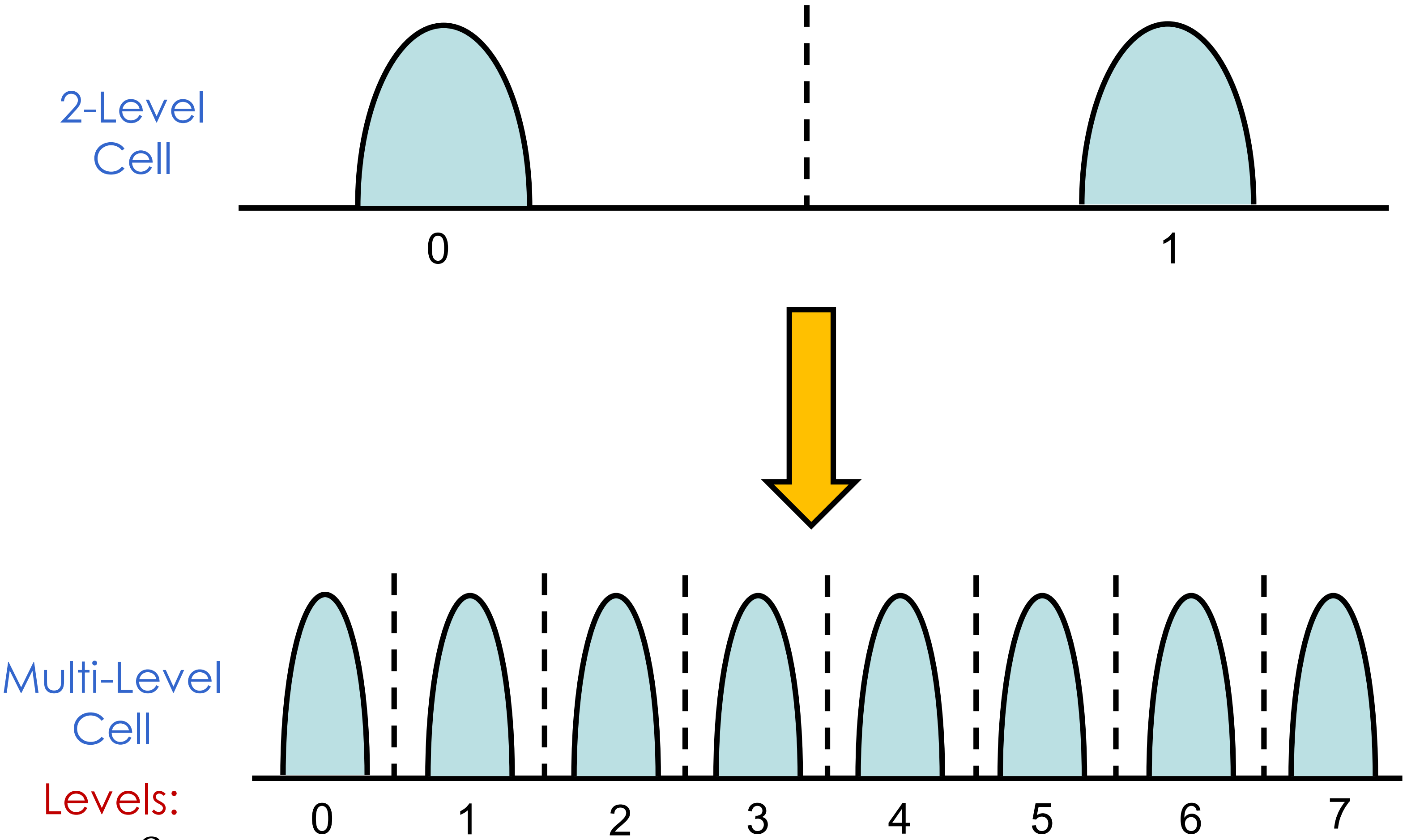
Multi Level NVM



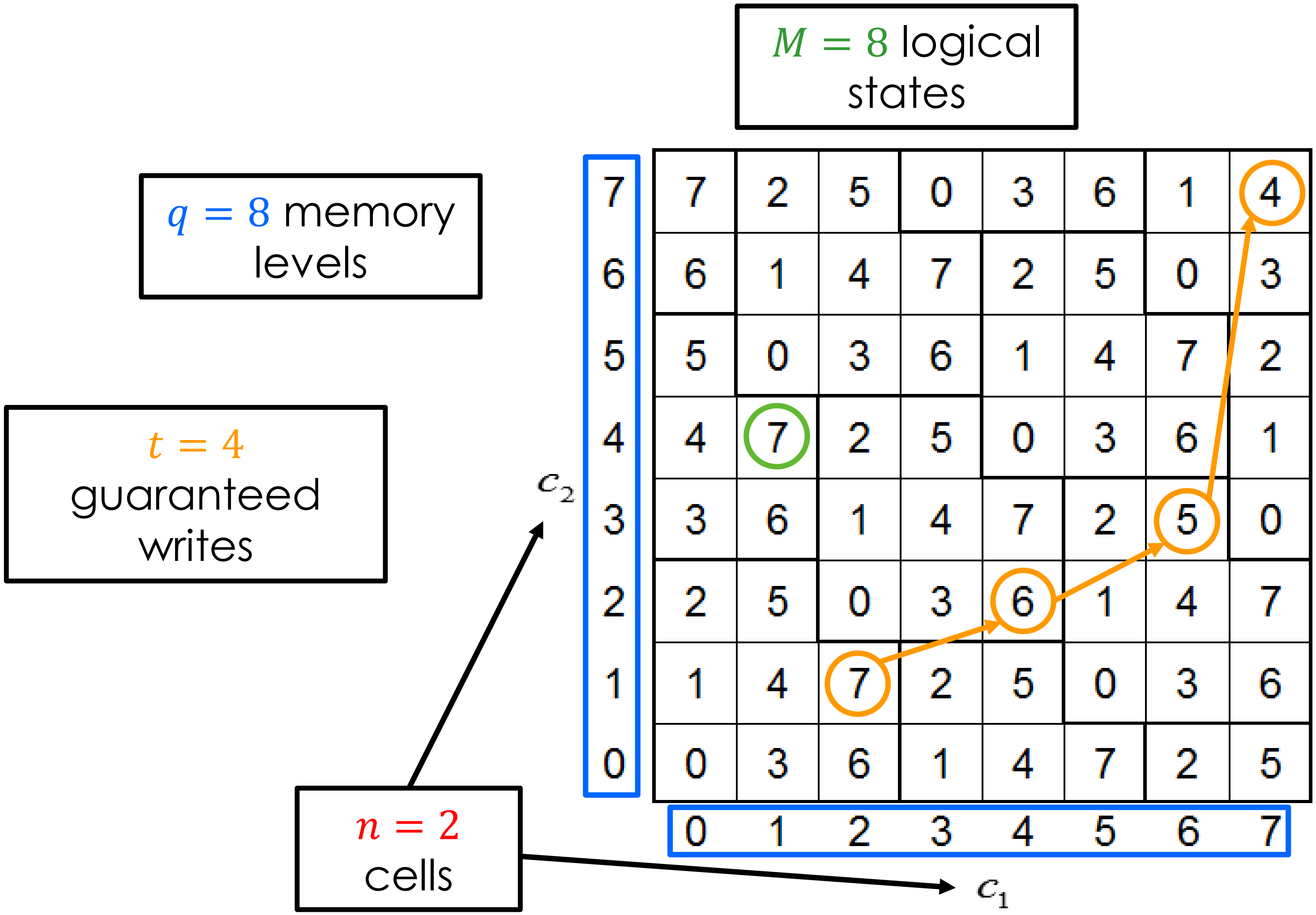
Levels:
$q = 8$

Writes in a Flash cell

- **The main problem of Flash storage:**
  - Cannot update data in place.
  - Major penalty in access speed and cell wear
- **Reason: cell levels cannot be <u>decreased</u> without erasing a large data unit.**
- **WOM codes: Mitigate this problem** [Rivest, Shamir ‘82]
  - Allow updating logical data multiple times, without decreasing physical cell levels
- **Potential application: special SSD “hot zone”**
  - File-system journals
  - FTL bypass, e.g. encryption keys, modem information

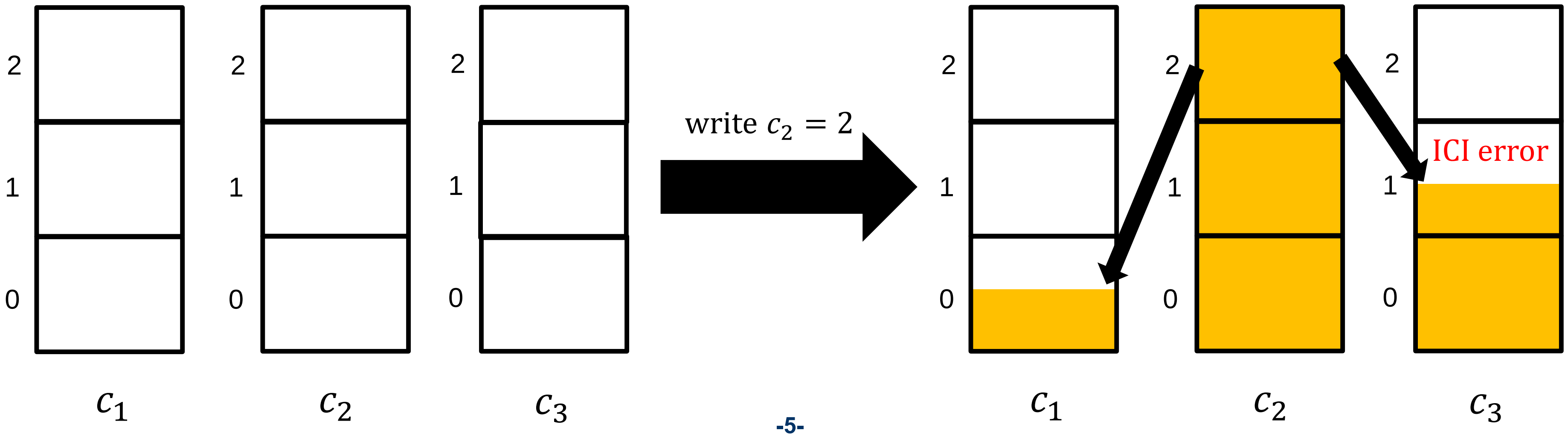
WOM code - Building Block

- **Example:** $(\boldsymbol{n = 2}, \boldsymbol{q = 8}, \boldsymbol{t = 4}, \boldsymbol{M = 8})$



Problem: Errors and WOM codes

- **Memory cells suffer read/write errors**
- **Re-writing further degrades reliability due to inter-cell interference (ICI)**
- **Combining WOM codes with error-correcting (EC) codes (ECC) is non-trivial**
  - we only have weak or non-practical ECC-WOM codes



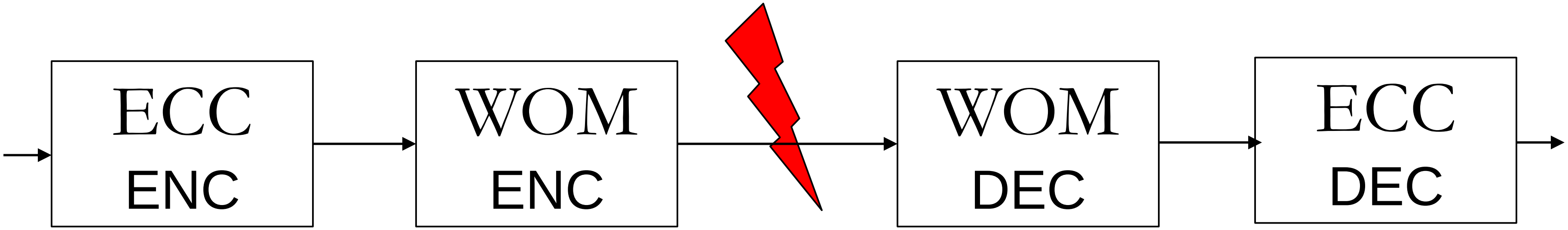
WOM-ECC joint design

- **Design practical error-correcting WOM codes**
  - Low redundancy
  - For common memory errors
    1. Guaranteed error correction
    2. Random errors
  - Any arbitrary number of errors τ

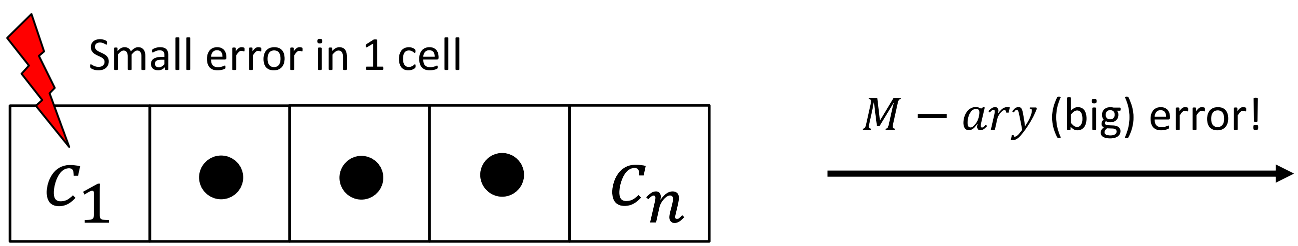
- **Prior work**
  - [Yaakobi, Siegl, Vardy, Wolf ’12], [Zemor, Cohen ‘91] – specific τ
  - [Jiang, En-Gad, Langberg, Bruck ‘13] – Polar codes, asymptotic block length

Challenge of combining WOM and ECC

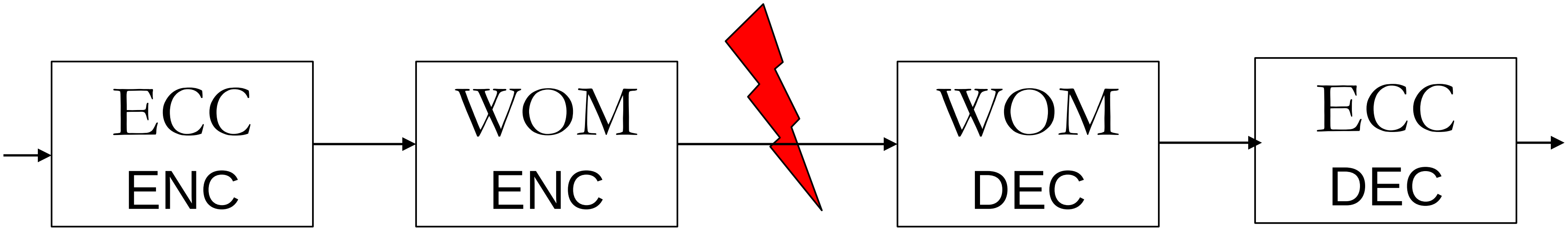
- **Forward concatenation:** **error propagation**



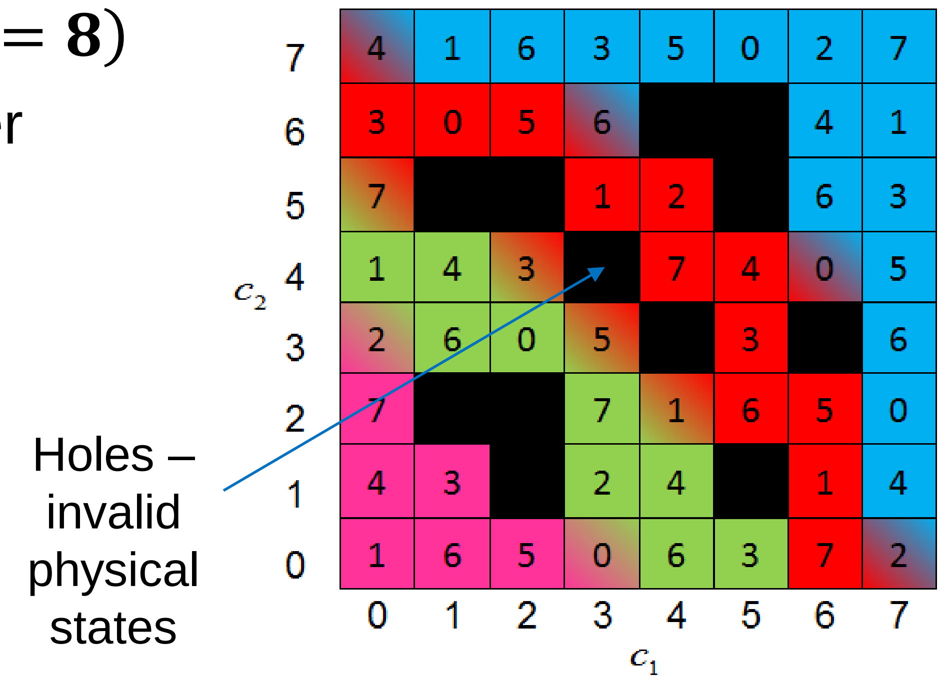
- **Example:** $(\boldsymbol{n}, \boldsymbol{q}, \boldsymbol{t}, \boldsymbol{M})$ **WOM**
  - Small errors in cell levels result in $M - ary$ errors
  - WOM is part of the channel



Concatenate ECC with special WOM codes

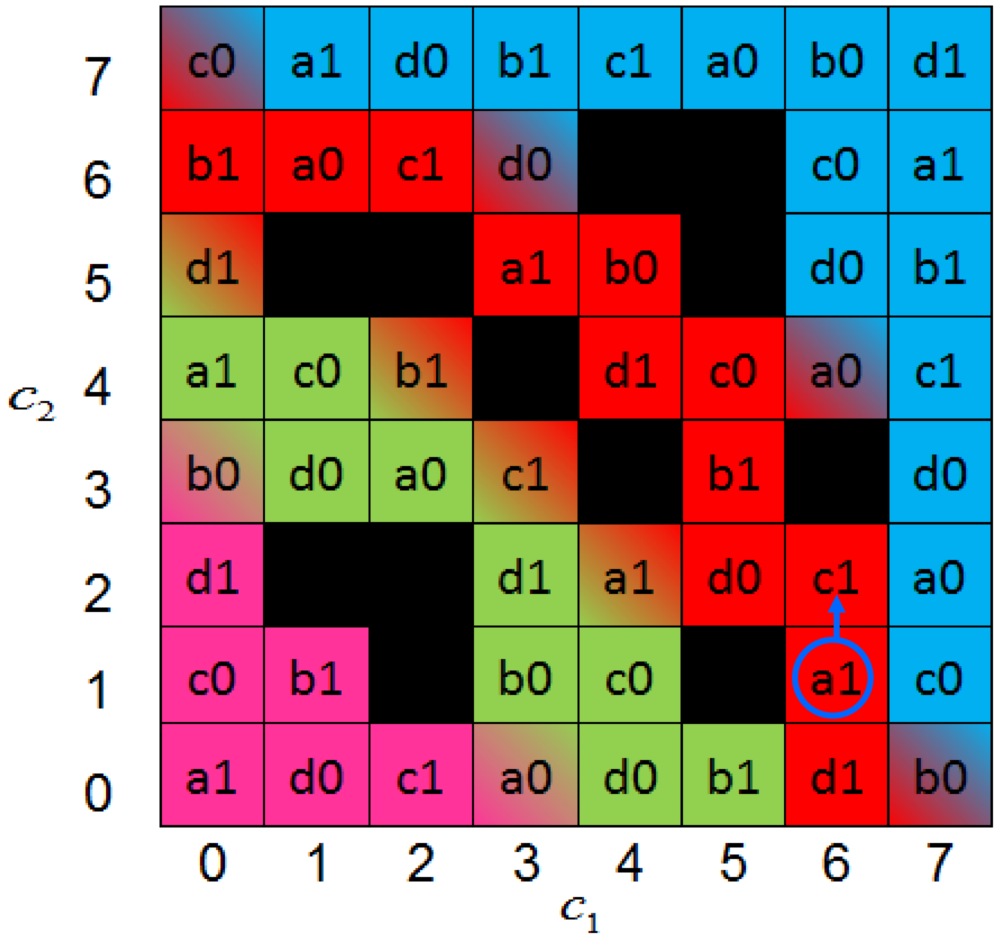


- ECC "sees" the WOM code as part of the channel
- Example of our special WOM codes
  - $(\boldsymbol{n = 2, q = 8, t = 4, M = 8})$
  - $k \in \{1,2,3,4\}$ write number
- Correction Guarantee!



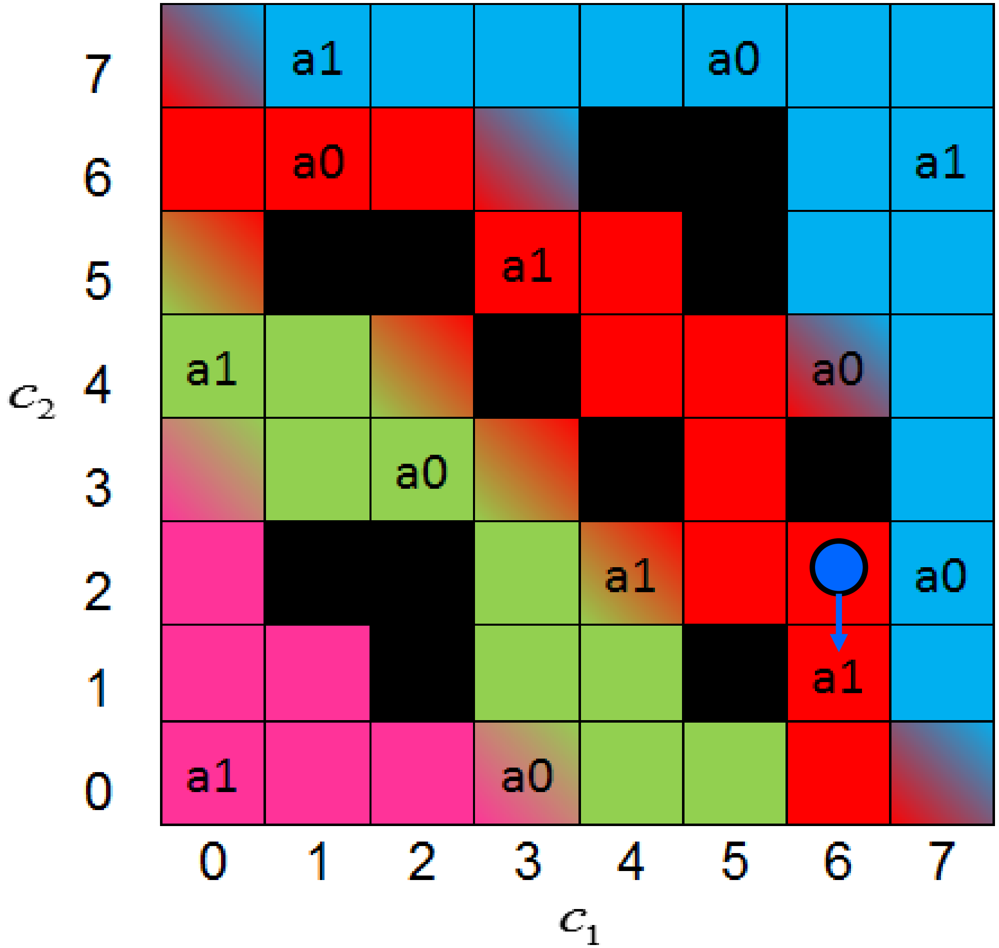
EC-WOM construction for mag-1 errors

- Mag-1 error results in $4-ary$ error or erasure
- First correct $4-ary$ errors



EC-WOM construction for mag-1 errors

- Decoder knows correct $4-ary$ symbol and write number
- Looks for closet $4-ary$ symbol within same write region



$4-ary$ symbol a

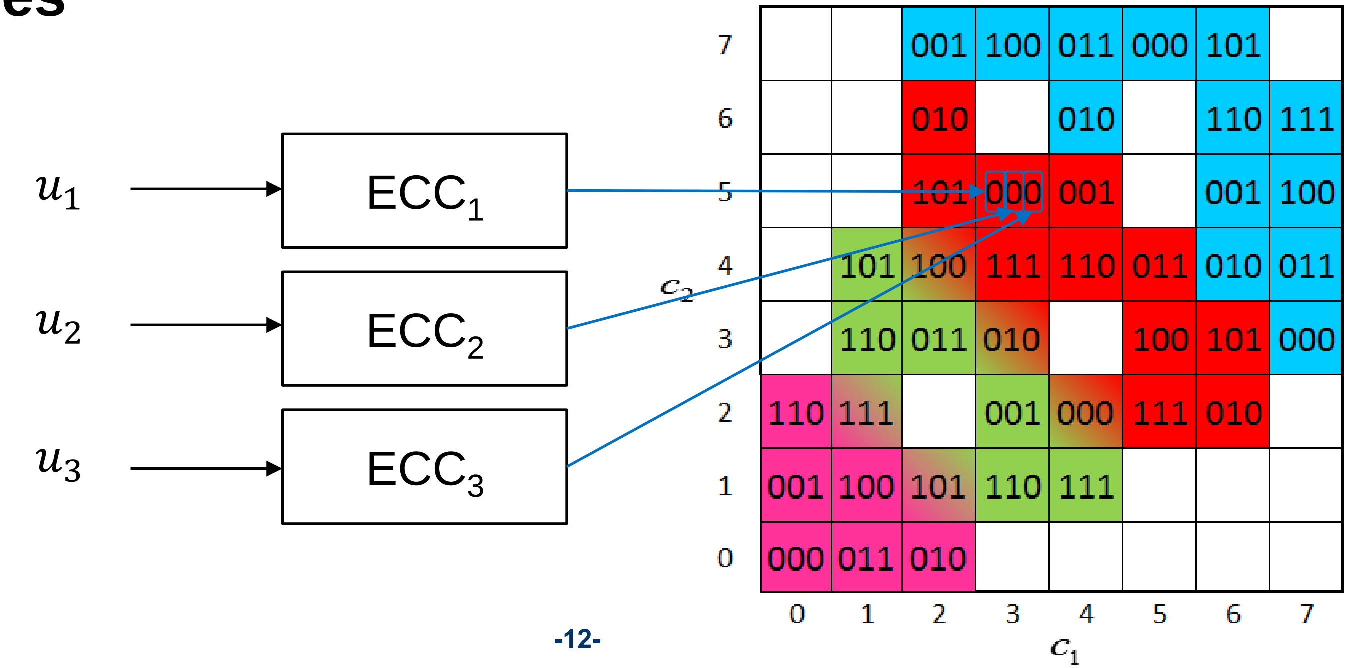
Comparison of Constructions

- **Closed form $\forall \tau, N$ (approximate)**

Construction	EC-Rate	Error Type	Alphabets
1	$1-\frac{\frac{7}{8}(2\tau-1)log_8(N)}{N}$	Any	8
2	$1-\frac{\left(\frac{3}{4}(\tau-1)+\frac{1}{2}(2\tau-1)\right)log_8 N}{N}$	mag-1	2,4
3’	$1-\frac{\frac{1}{4}(6\tau_1+10\tau_2-5)log_8 N}{N}$	τ_1 single mag-1, τ_2 general	4,2
4	$1-\frac{\frac{3}{2}(3\tau-2)log_8 N+2\tau-1}{N}$	amag-1	2,2

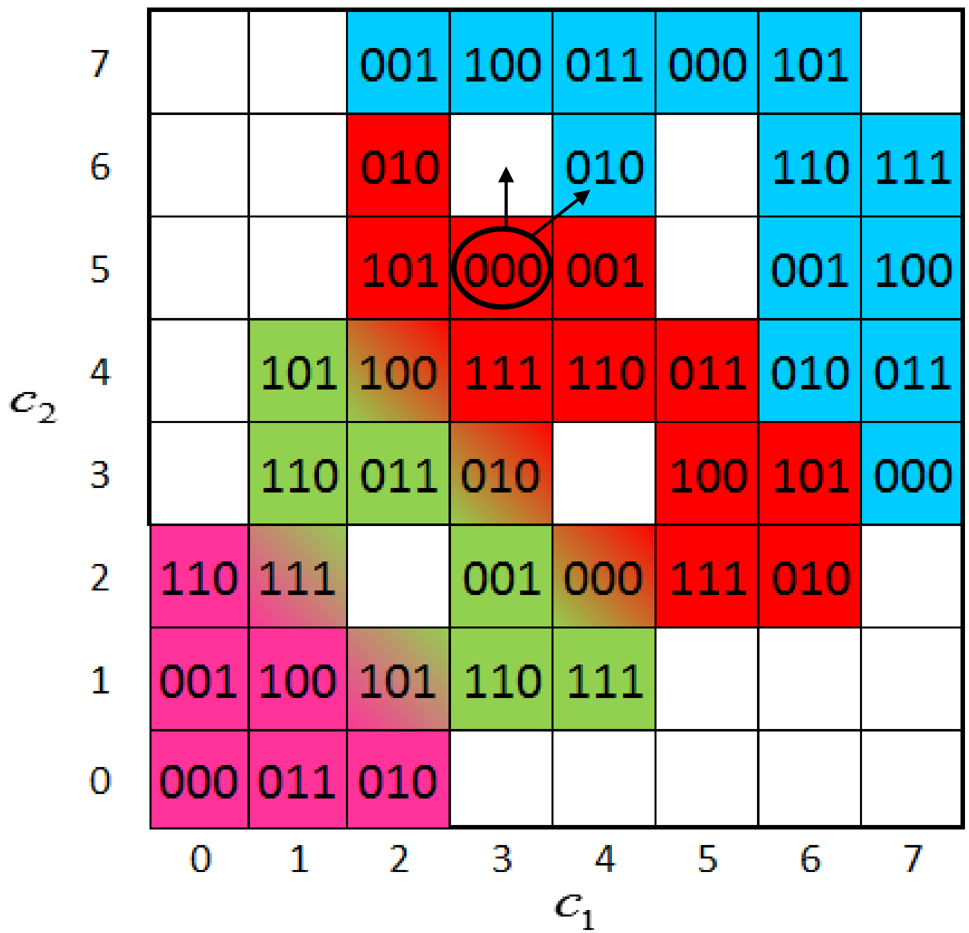
EC-WOM codes for random errors

- **<u>Objective</u> – improve and analyze WOM error correction over realistic memory channels**
- **Multi level coding** [Wachsmann, Fischer, Huber '99]
- **High order symbols are mapped to multiple bits**
- **Main idea: propagate information from low to high bits**
- **Multi-stage decoding, WOM converts many errors to erasures**



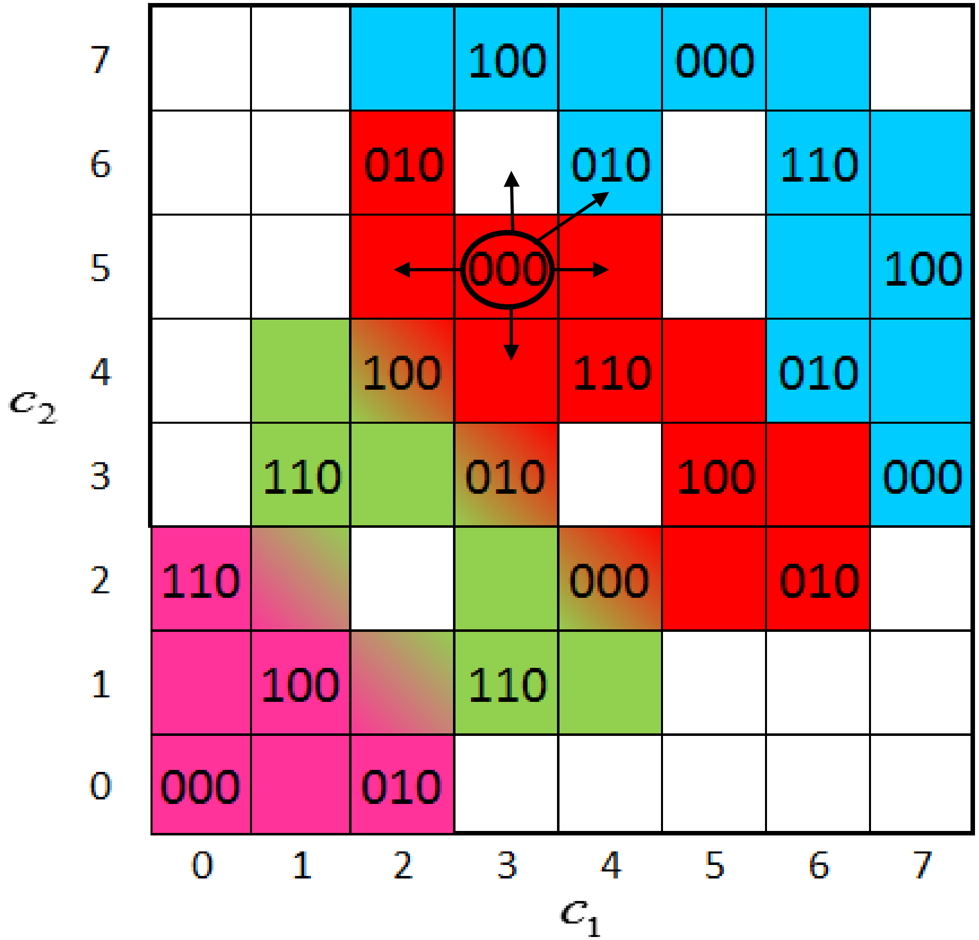
Erasures

Erasures: write number $k = 3$, bit number 1 $i = 1$ (LSB)



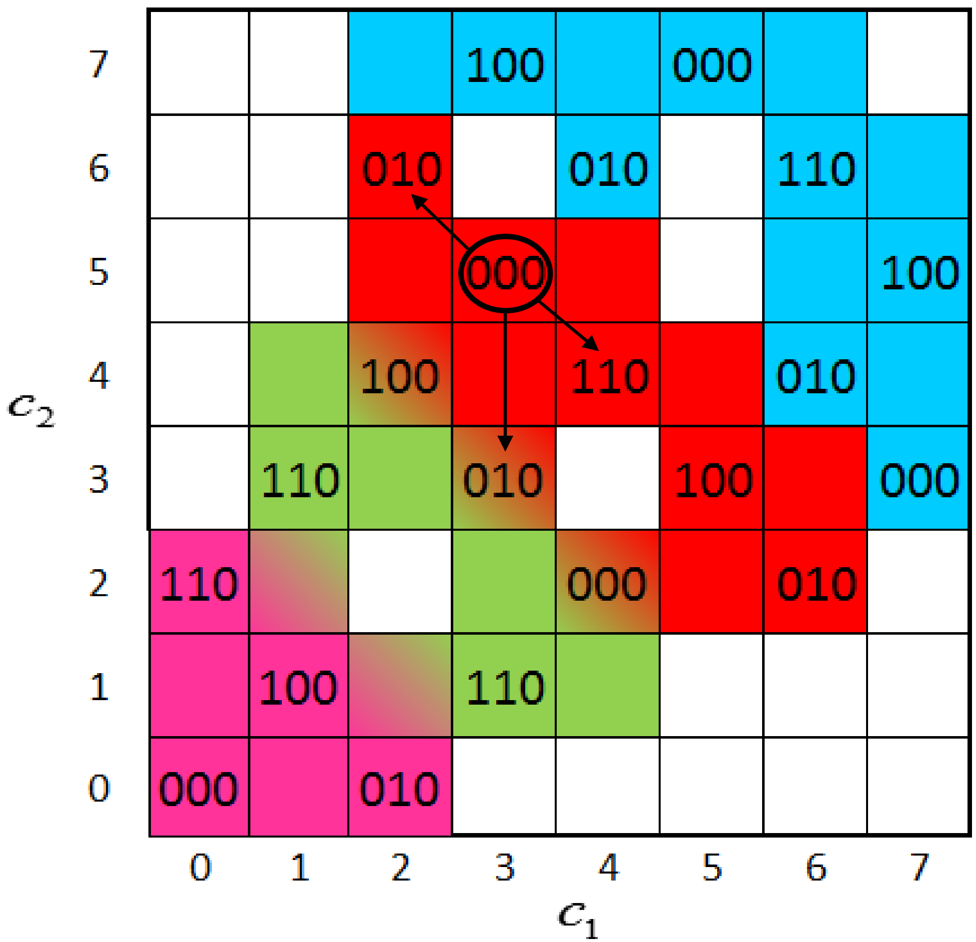
Erasures

Erasures: **write number** $k = 3$**, bit number** $i = 2$**, first bit** $b_1 = 0$



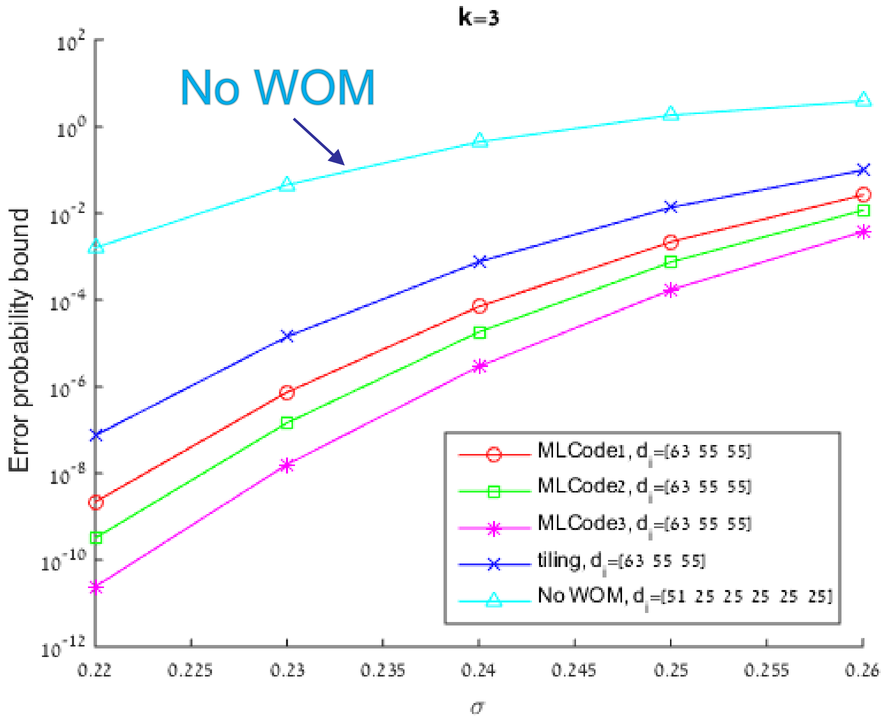
Errors

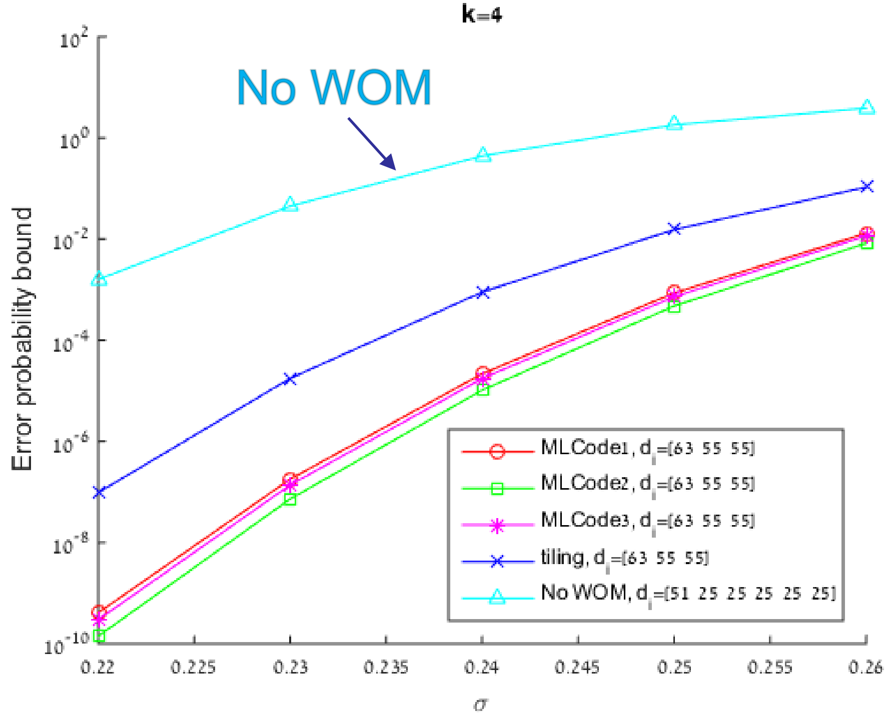
Errors: write number $k = 3$, bit number $i = 2$, first bit $b_1 = 0$



Error probability bound – AWGN example

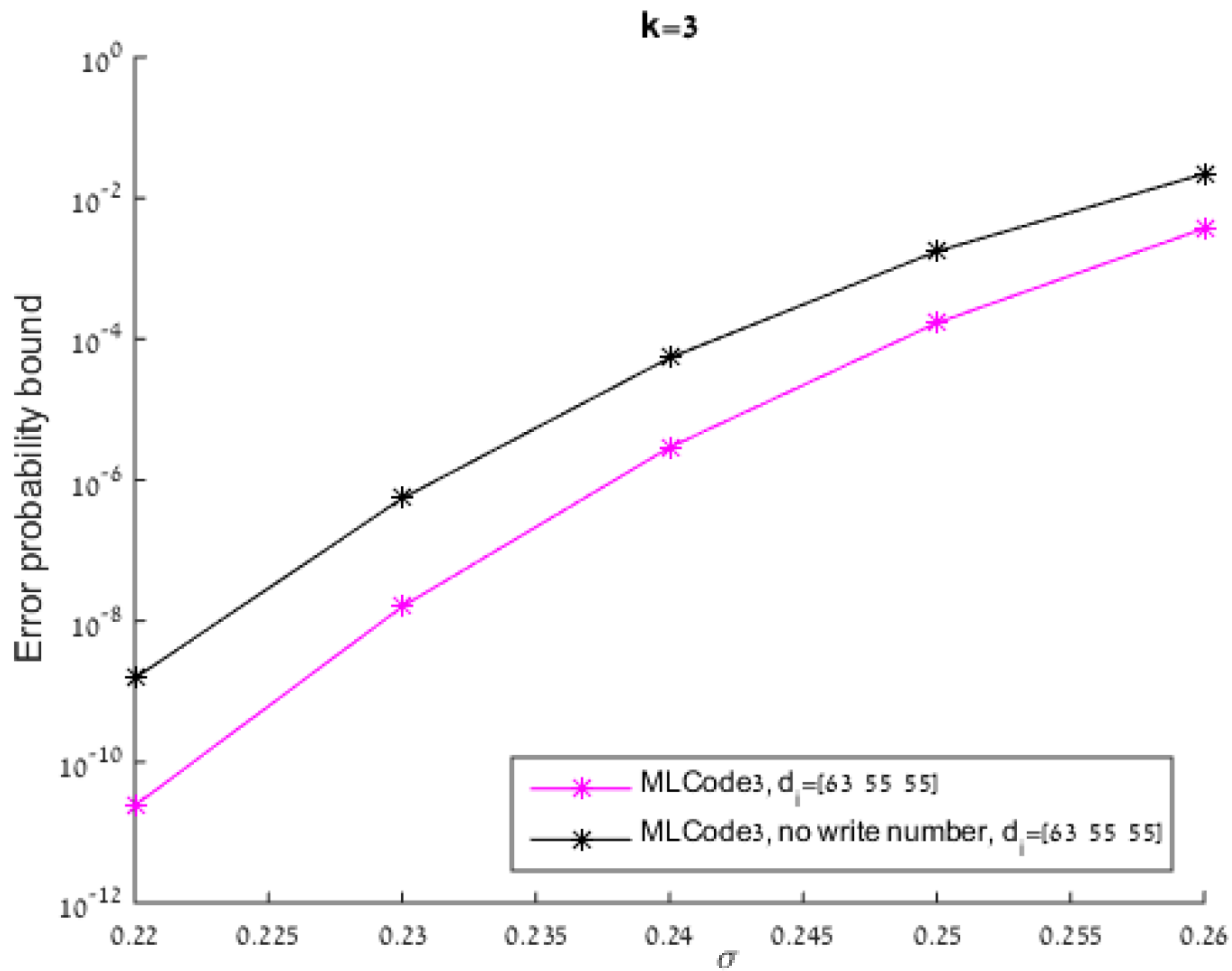
- AWGN analysis, standard deviation σ
- Optimized redundancy distribution
- Code performance changes with write number k
- Propagation Index can predict WOM performance





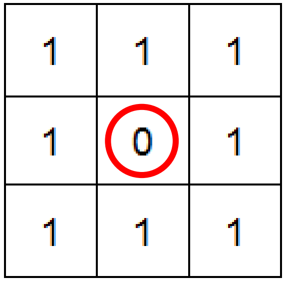
Knowing the write number helps

- **Comparison between decoders with/without write-number information**

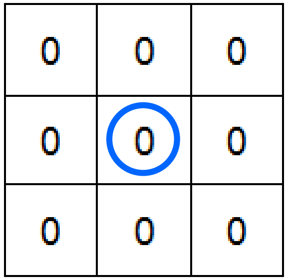


Multi level coding soft decoding

- **Channel output has additional reliability information**
- **Use soft BCH decoders: GMD** [Forney ‘73]
- **Reliability 1 – Certain bit**
- **Reliability 0 – Erasure**
- **Soft information with hard memory reads**



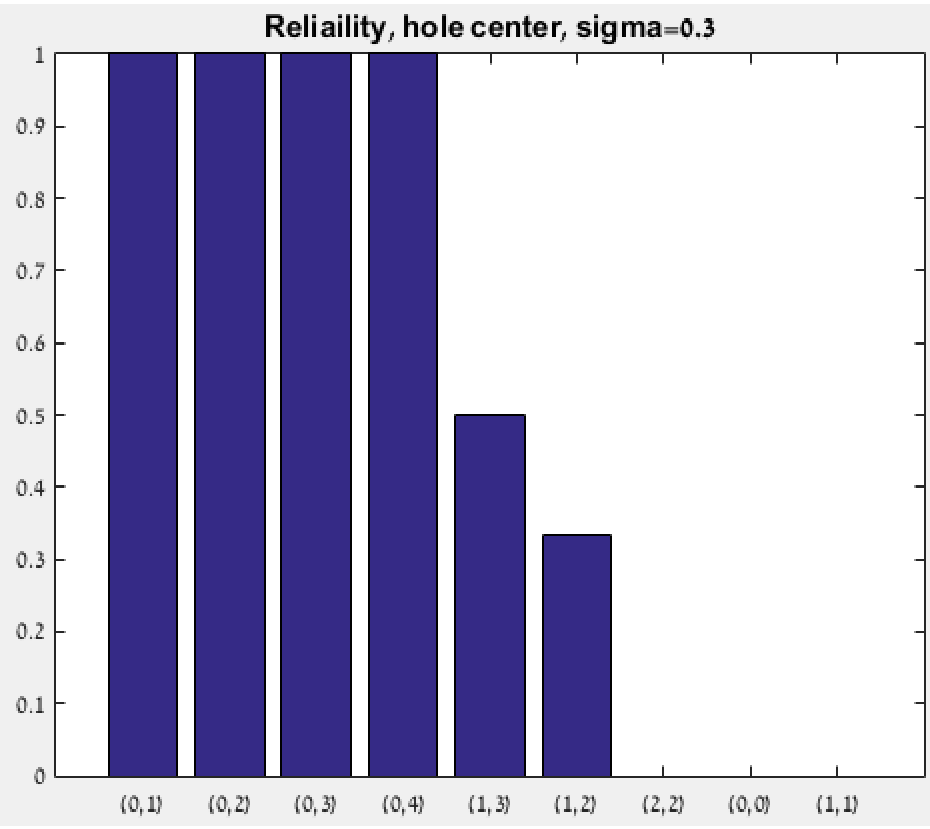
Low reliability

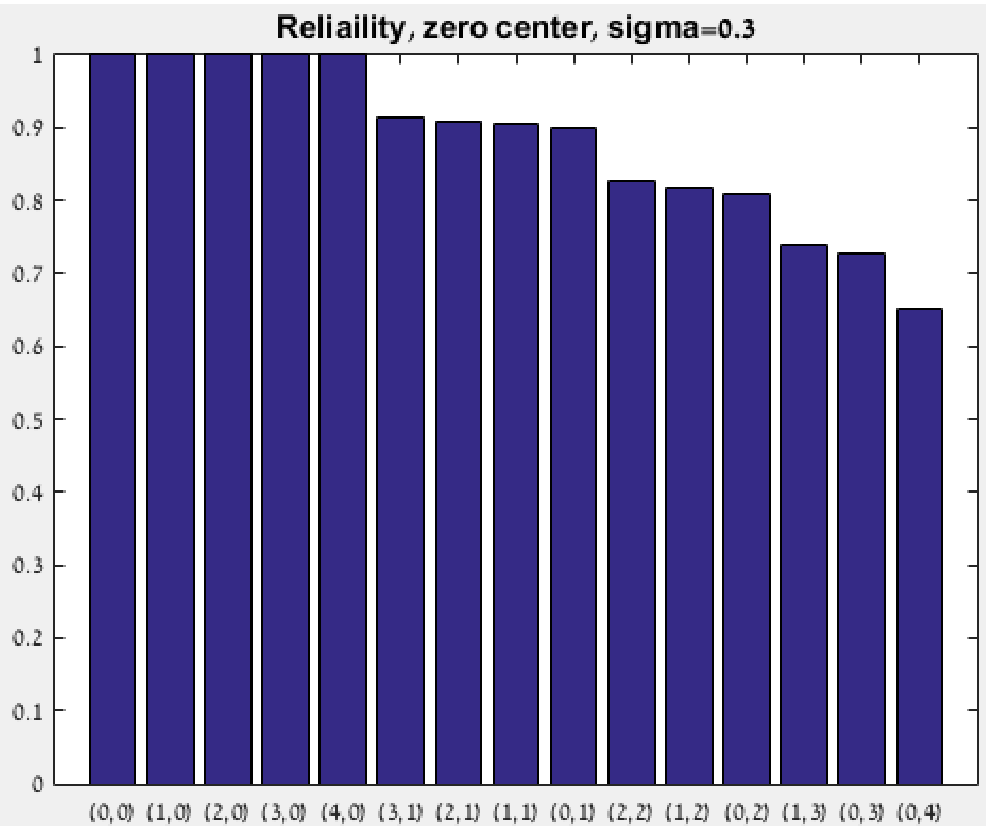


High reliability

Multi level coding soft decoding - reliability

- Problem: order reliabilities of each physical state
- AWGN: *non* mag-1 errors are unlikely
- (m, n): m adjacent zeroes, n adjacent ones
- Lemma: order of reliabilities does not change for $\sigma < 0.341$





Publications

- A. Solomon and Y. Cassuto, “Error-Correcting WOM Codes: Concatenation and Joint Design”, IEEE Transactions on Information Theory, 09/2019.
- A. Solomon and Y. Cassuto, “Error-Correcting WOM Constructions through Concatenation and Joint Design”, IEEE International Symposium on Information Theory, ISIT 2018.

Summary

- **WOM codes can help flash based memories**
- **Joint design of WOM and ECC codes improves reliability**
  - Guaranteed error models
  - Random error models
- **Also improves decoding complexity**
- **Can obtain soft information with hard memory reads**