LDPC Error Floor Prediction using Trapping Set aware Code Shortening

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Outline

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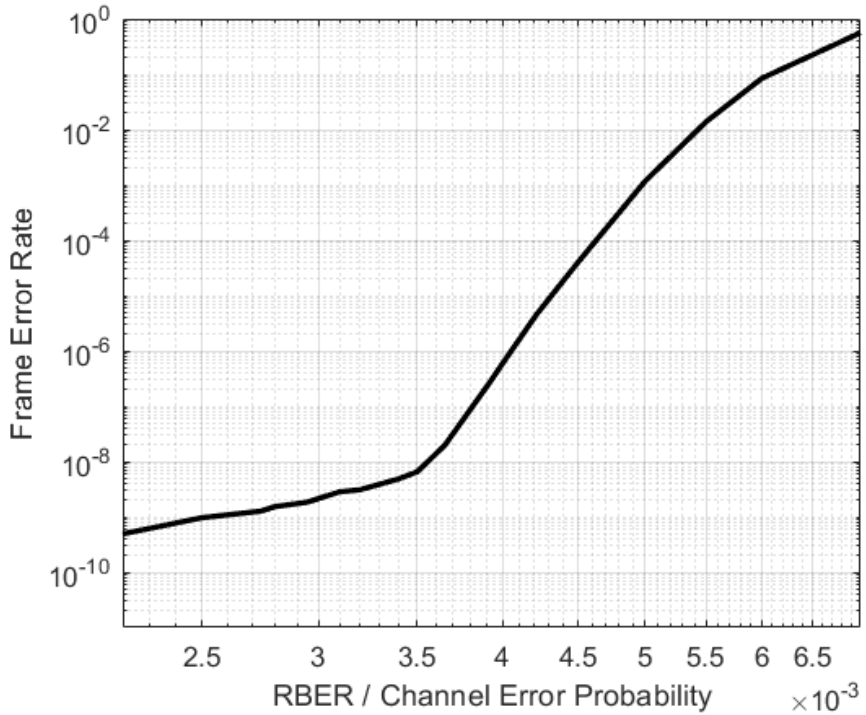
Error floor Problem in LDPC Decoders

Tom Richardson (Allerton 2003)

- An abrupt degradation of FER at low RBER caused by a failure of an iterative decoder to converge to a codeword
- Error floor is attributed to dense subgraphs present in the Tanner graph : Trapping Sets $\tau(a, b)$
- $\tau(a, b)$: a set of not eventually correct variabe nodes of size a, inducing a subgraph of b odd degree check nodes.



Error floor Problem in LDPC Decoders

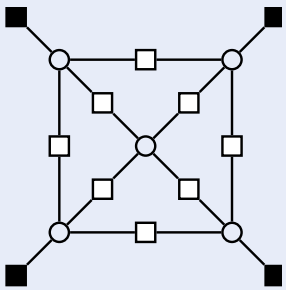
Vasic (Allerton 2005, ICC 2006)

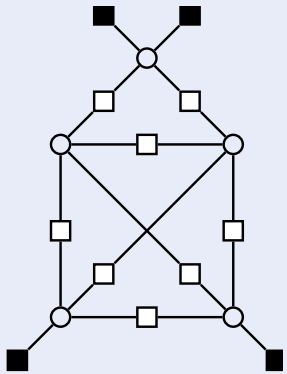
- The harmfulness of Trapping Sets is based on uncorrectable error patterns on isolated Trapping Sets.
- **Critical Number of a Trapping Set : c_τ**

  minimum number of errors in $\tau(a, b)$ (out of a) which causes failure, when $\tau(a, b)$ is **isolated from the rest of the Tanner graph**
- **Strength of a Trapping Set : s_τ**

  number of error patterns of weight c_τ bits, for which the decoder fails on the **isolated** $\tau(a, b)$

Two examples of $\tau(5, 4)$ Trapping Sets





Outline

Major Flaw of Existing Trapping Set Characterization

Subgraphs that are "believed" to be harmful

- Whether a Trapping Set is harmful depends on **a decoder** and its **neighborhood** in the Tanner graph
- For simple decoders (BF, Gal-B), Trapping Sets can be treated isolated from the rest of the graph
- For stronger decoders (min-sum, FAID), an isolated Trapping Set is not sufficient to predict its impact on error floor
- Trapping Sets with the same values of a and b can be **either** *harmful* **or** *not harmful*

We propose a new characterization of harmfulness to take into account

1- The **neighborhood** of the Trapping Set within a particular QC-LDPC code,

2- The **particular message passing decoding** rules $\phi = (\phi_V, \phi_C)$ that are used for decoding.

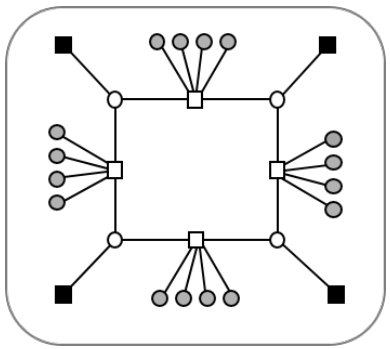
Advantages of our approach

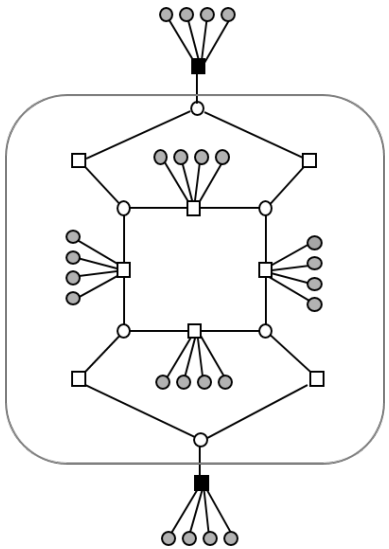
- The harmfulness of $\tau(a, b)$ will depend on the sparseness and the **topology of its neighborhood**.
- By considering a particular decoding rule, we will consider structures that are harmful *only* **for this particular decoder**
- We do not consider that a TS can be "universally" harmful, as it is often assumed in existing works.

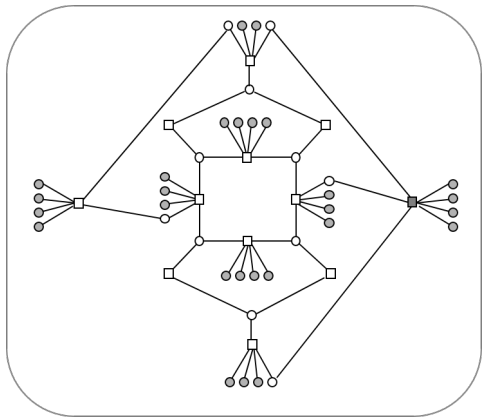
New Characterization of Harmfulness : expansion-contraction procedure

Step 1 : Expansion (neighborhood dependent)

- Let us consider a small Trapping Set $\tau(a, b)$ (not necessarily harmful) - or a single cycle - in a given LDPC code,
- The trapping set $\tau(a, b)$ is expanded by adding data node neighbors to the graph as long as they create **new cycles**,
- The expansion is recursively repeated until no new cycles can be created,
- After expansion, we obtain a larger structure $\mathcal{T}(A, B)$, with more cyles than the original TS,



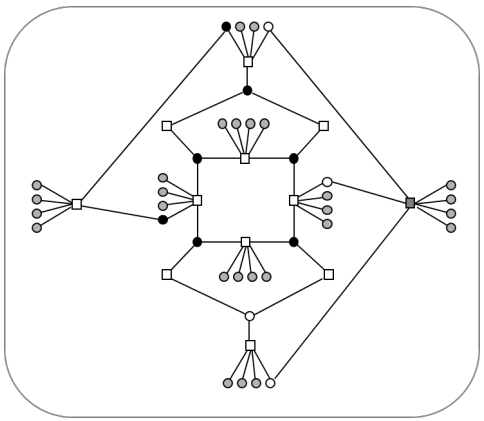


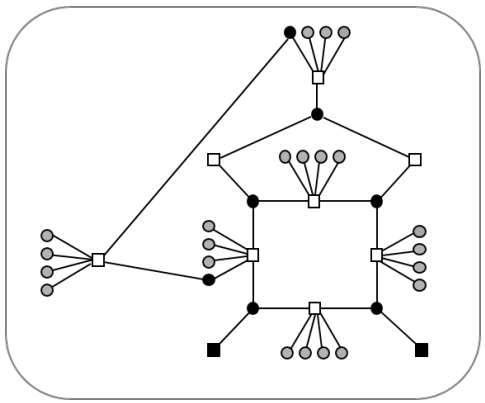


New Characterization of Harmfulness : expansion-contraction procedure

Step 2 : Contraction (decoder dependent)

- We consider the expanded Trapping Set $\mathcal{T}(A, B)$, and compute its critical number $\mathbf{c}_{\mathcal{T}}$ and strengh $\mathbf{s}_{\mathcal{T}}$,
- Let $\{\mathbf{e}_1, \ldots, \mathbf{e}_s\}$ be the set of error patterns of weight $c_{\mathcal{T}}$ (indicated in black circles),
- Define $\mathcal{S}$ as the union of the support of all error patterns : $\mathcal{S}\left(\mathbf{e}_1, \ldots, \mathbf{e}_s\right) = \bigcup_{k=1}^{s} \mathcal{S}(\mathbf{e}_k)$
- $\mathcal{S}$ is composed of all the bits in $\mathcal{T}(A, B)$ which participate in *at least* one decoding failure.





Definition

The subgraph corresponding to $\mathcal{S}$ is declared as **the harmful TS**, *denoted* $\tau_h\,(a_h, b_h)$

Harmfulness Spectrum of a LDPC Code

List of Harmful Trapping Sets

- Let $\mathcal{T} = \left\{ \tau_{h_1}, \tau_{h_2}, \ldots, \tau_{h_T} \right\}$ be the set of harmful Trapping Sets selected with expansion-contraction procedure
- $\mathcal{T}$ is identified from **large Trapping Sets** in an actual QC-LDPC code, *i.e.* not isolated from their neighborhood,
- $\mathcal{T}$ is identified by decoding error patterns using a particular message passing decoder, *i.e.* the set $\mathcal{T}$ differs from one decoder to another.

Harmfulness Characterization

- For each $\tau_{h_i} \in \mathcal{T}$, we consider its harmfulness as $\left(\mathbf{c}_{\tau_h}, \mathbf{s}_{\tau_h} \right)$
- Structures with the **smallest critical number** c_{τ_h} are the *most harmful*
- Within the structures with same $\mathbf{c}_{\tau_h}$, the ones with **largest strengh** $\mathbf{s}_{\tau_h}$ are the *most harmful*
- Relative harmfulness of two trapping sets with same critical numbers is the ratio of their strengths

Harmfulness Ranking

Using our expansion-contraction procedure, we obtain an **exact ranking** *of the harmful Trapping Sets, for a given LDPC code, and a given decoder.*

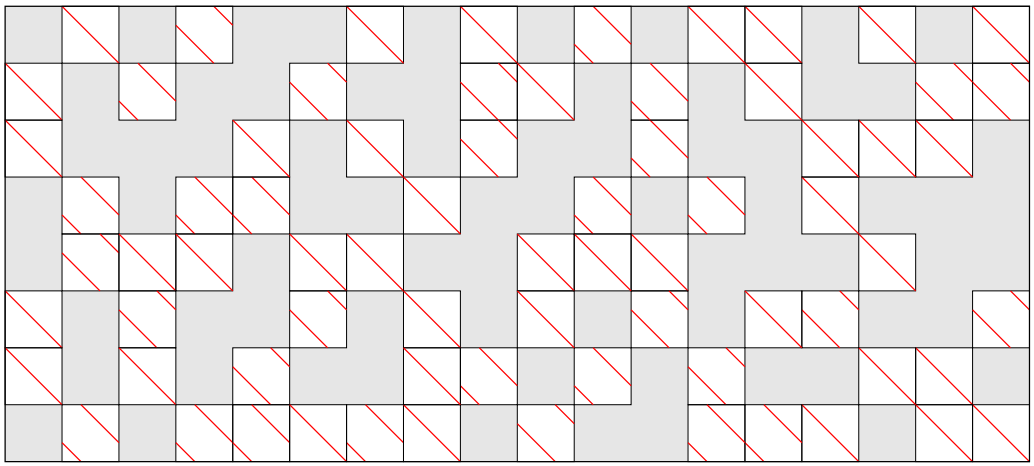
Outline

Shortening procedure

- Regular Quasi-cyclic LDPC (QC-LDPC) code defined by its PCM $\mathbf{H}$, organized in N_b *block-columns*,

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{:,1} & \mathbf{H}_{:,2} & \dots & \mathbf{H}_{:,N_b} \end{bmatrix}$$

- $\mathbf{H}_{:,i}$ contains d_v circulant blocks, and $(M_b - d_v)$ all zero blocks,
- Select s block-column indices $\mathbf{i} = [i_1, \dots, i_s]$, with $M_b < s < N_b$
- Shortening = Extracting from $\mathbf{H}$ the corresponding block columns.
- The shortened code $\mathbf{H_{short}} = \begin{bmatrix} \mathbf{H}_{:,i_1} & \mathbf{H}_{:,i_2} & \dots & \mathbf{H}_{:,i_s} \end{bmatrix}$ has rate $R_{short} = 1 - M_b/s < 1 - M_b/N_b$.



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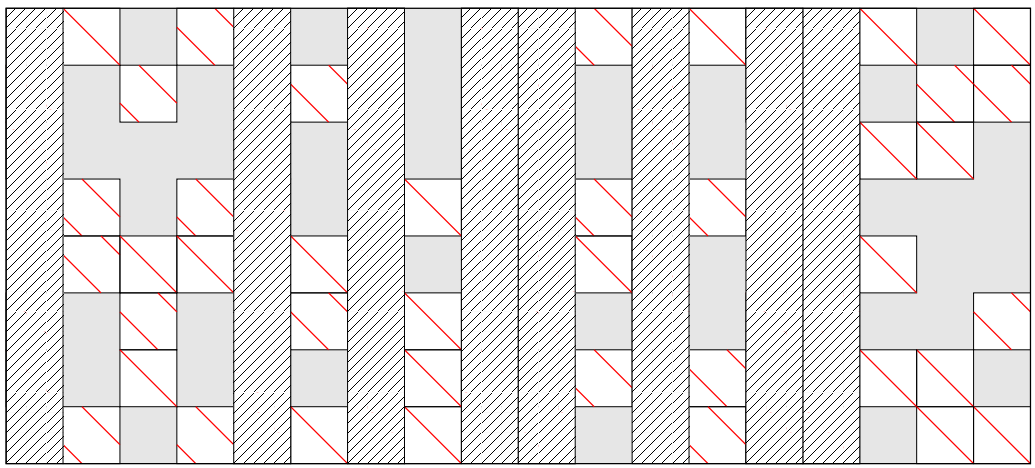

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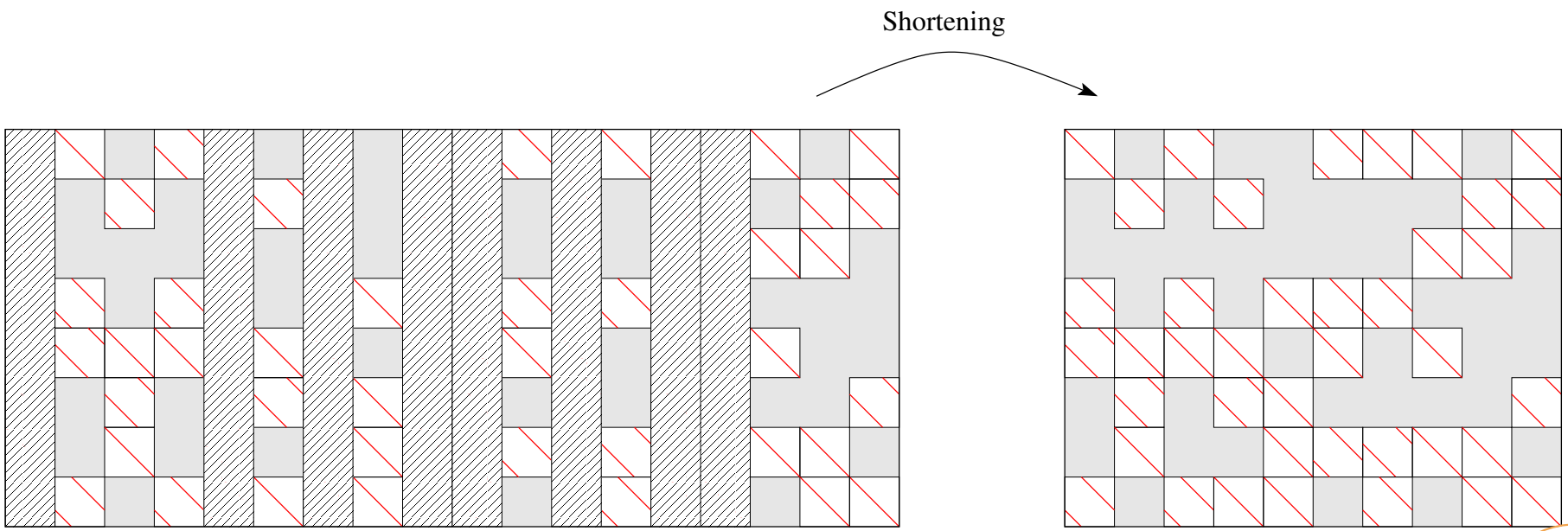


Code Shortening to Estimate Error Floors

Design the **worst possible** shortened code

- Build a shortened code $\mathbf{H}_{\mathbf{short}}$ which contains the same most harmful Trapping Sets as the original code $\mathbf{H}$, and run Monte Carlo simulations on $\mathbf{H}_{\mathbf{short}}$ to detect its error floor
- If the decoder fails on the same structures for $\mathbf{H}_{\mathbf{short}}$ and $\mathbf{H}$, both curves will have an error floor **with the same slope**, but since $\mathbf{H}_{\mathbf{short}}$ has lower rate, the error floor will appear at a higher FER, resulting in computational savings
- Chosing properly the shortened columns is critical for the efficiency of our approach.
  - ◇ When s is large, close to N_b, it is easier to ensure that the harmful TSs of $\mathbf{H}$ will remain in $\mathbf{H}_{\mathbf{short}}$, but computational saving would be small
  - ◇ With a too small value of s, $\mathbf{H}_{\mathbf{short}}$ might not contain anymore the harmful TSs, resulting in an erroneous prediction of the error floor
- Our strategy is then to minimize the number of block-columns kept, while still ensuring that the harmful TSs are present in $\mathbf{H}_{\mathbf{short}}$
- This optimization problem is closely related to the well studied weapon-target assignment problem and the hypergraph demand matching problem

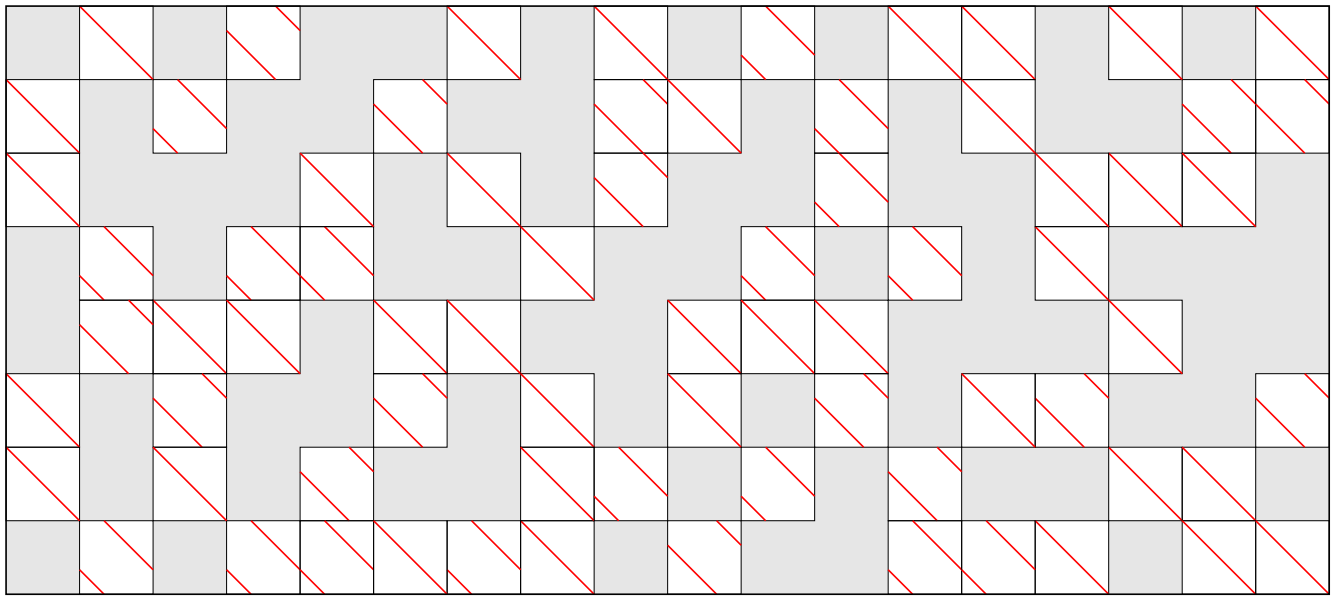
Design the **worst possible** shortened code

- We propose a pragmatic and simple approach to perform the shortening optimization

Target QC-LDPC Code

Example of LDPC code with Nb=18 columns and Mb=8 rows

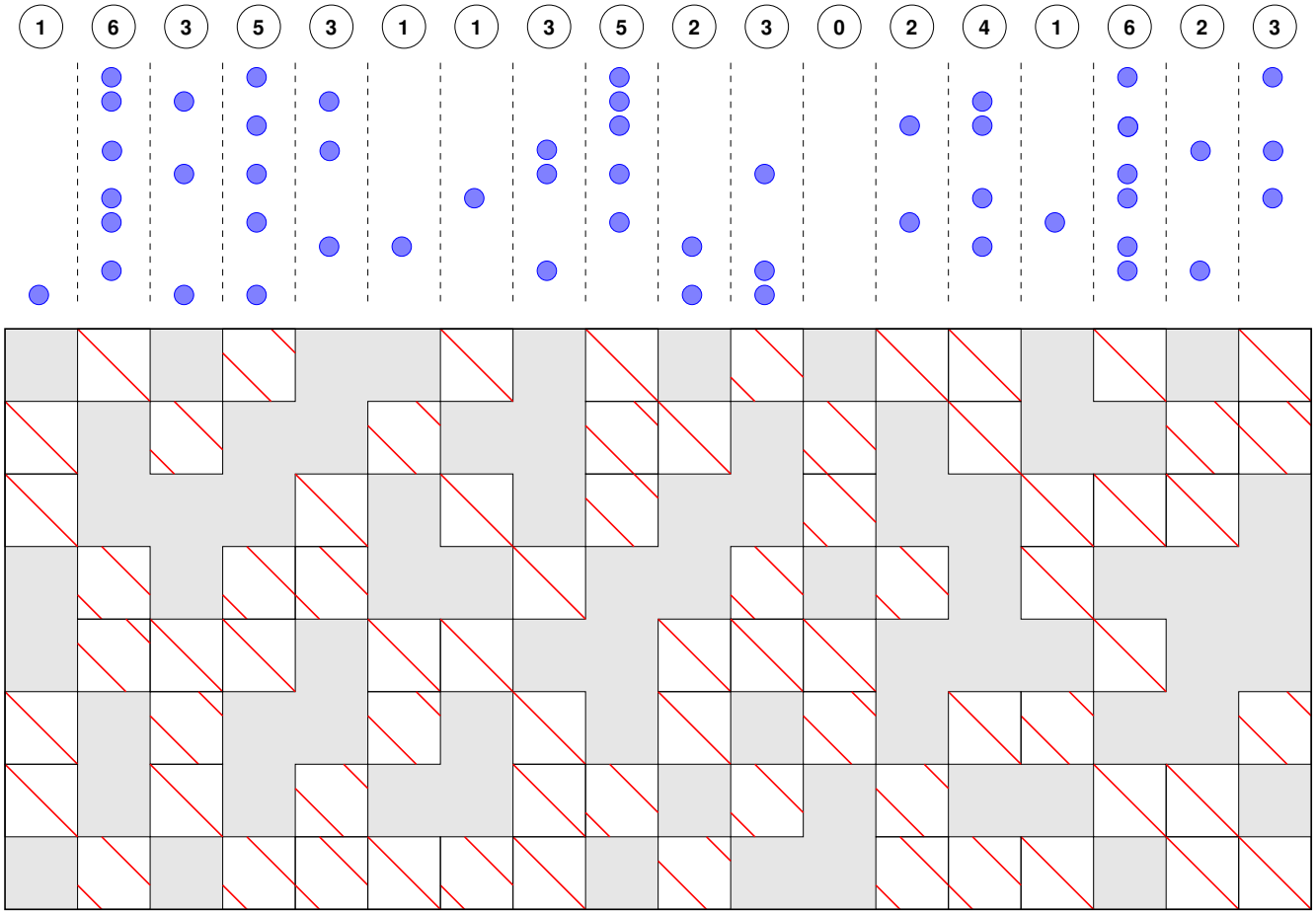
- Compute for this code a list of the $K = 10$ most harmful TS, with the expansion/compression method



Shortening STEP-1 : Accumulate Trapping Sets Indices

Shortening STEP-1

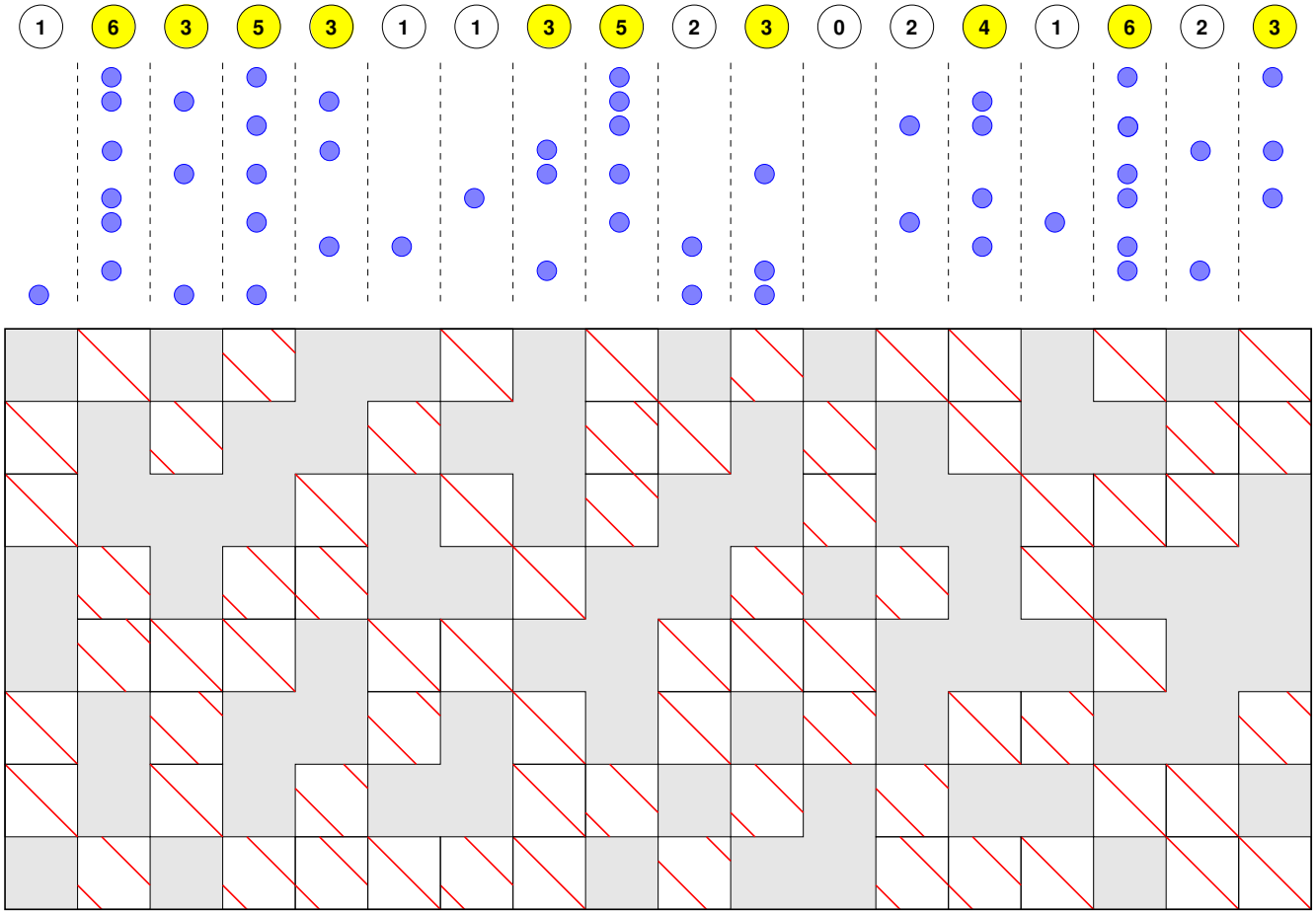
- The *i*-th block-column harmfulness = sum of harfmulnesses of all Trapping Sets having a variable node in it.



Shortening STEP-2 : Select the Worse Columns

Shortening STEP-2

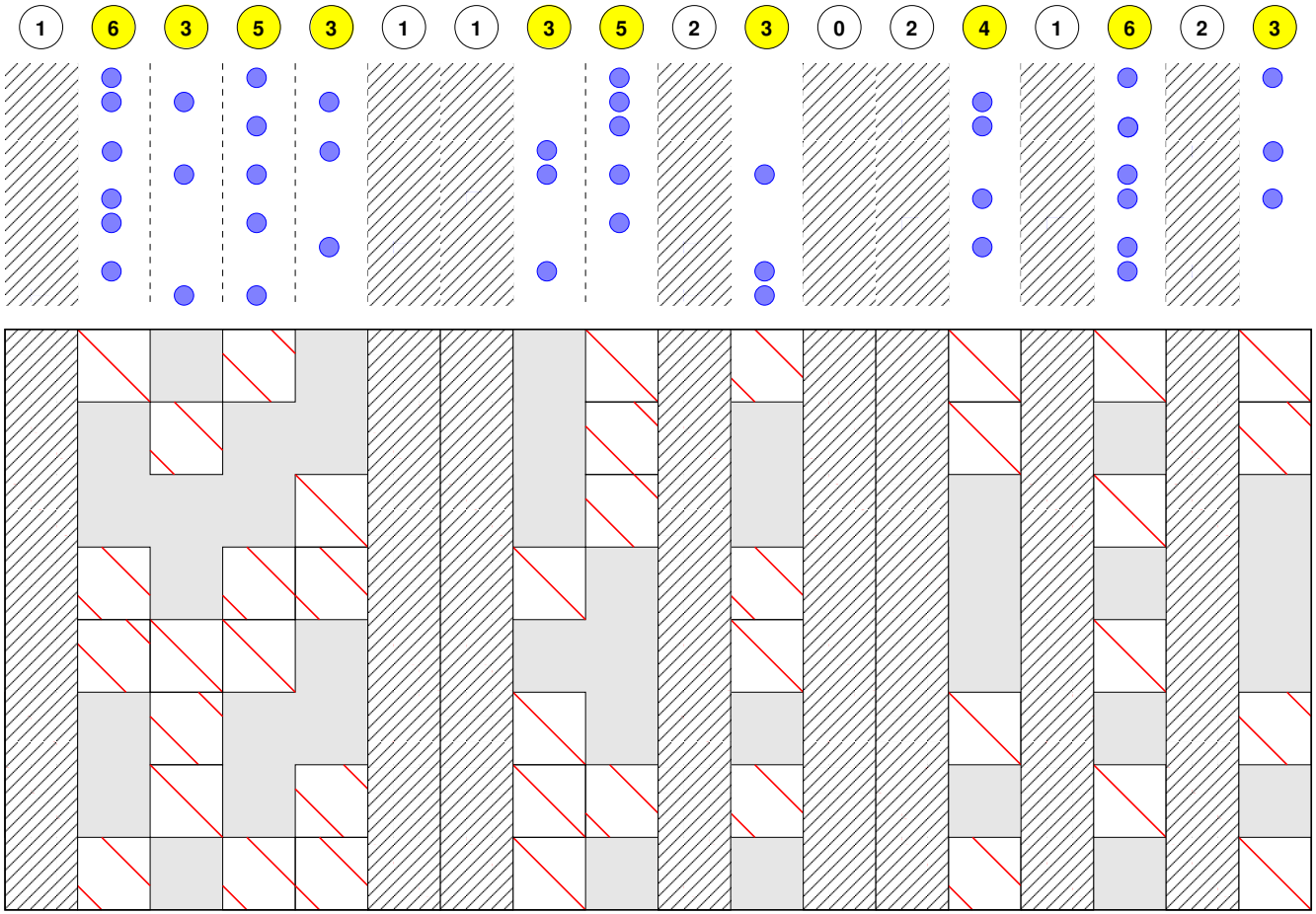
- Select the block-columns which have total harmfulness greater than a threshold t.



Shortening STEP-3 : Shorten Best Columns

Shortening STEP-3

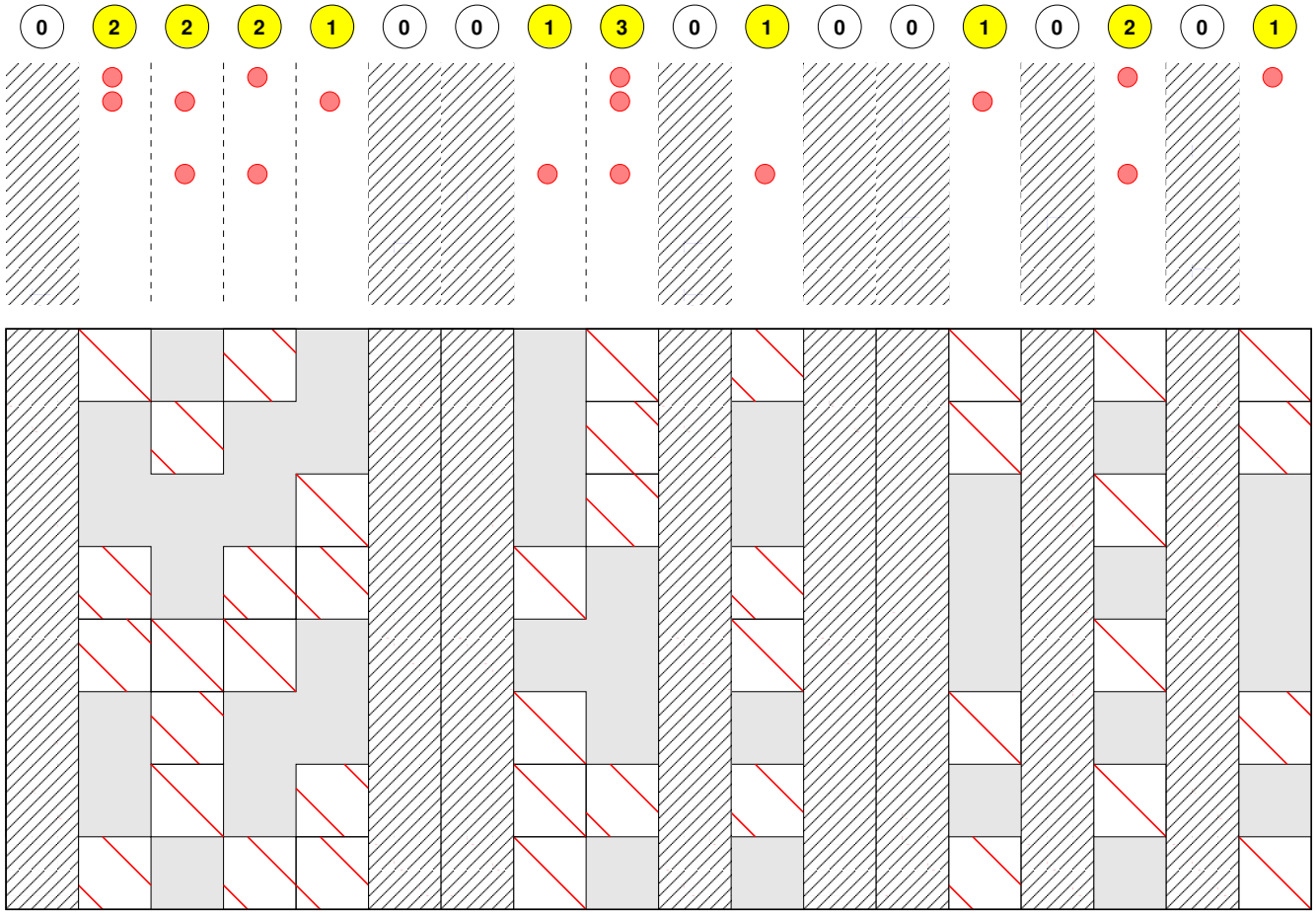
- Build a shortened version of the code, with Nbs=9 block-columns



Shortening STEP-4 : remaining harmfulness in H_{short}

Shortening STEP-4

- **Correction factor** - the ratio between remaining harmfulness weight and total harmfulness weight.

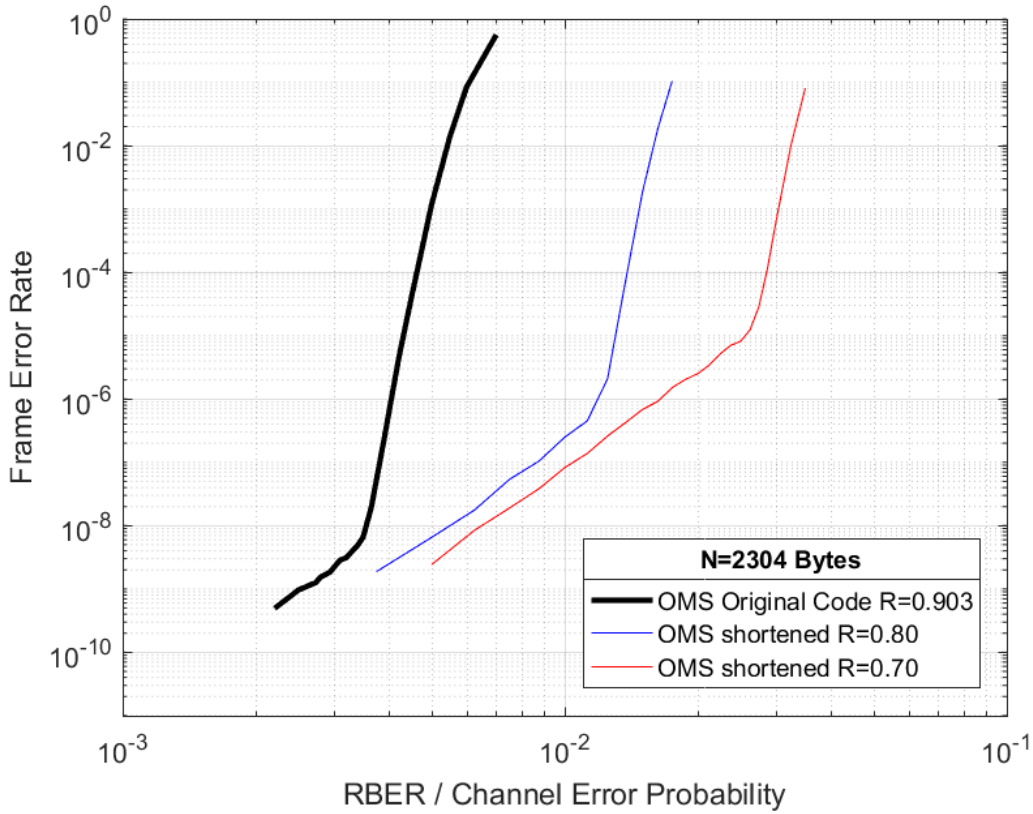


Outline

Results from Direct Simulation of the Shortened Codes

Target QC-LDPC code : N=2kBytes - $R = 0.903$ - $(M_b, N_b) = (56, 576)$ - $L = 32$

- Shortened Code C_1 : $(M_b, N_b) = (56, 283)$ - $R = 0.802$
- Shortened Code C_2 : $(M_b, N_b) = (56, 182)$ - $R = 0.692$
- **Decoder :** Offset corrected Min-Sum / 4 bits precision / 20 iterations



Results after Correction Factor

Target QC-LDPC code : N=2kBytes - $R = 0.903$ - $(M_b, N_b) = (56, 576)$ - $L = 32$

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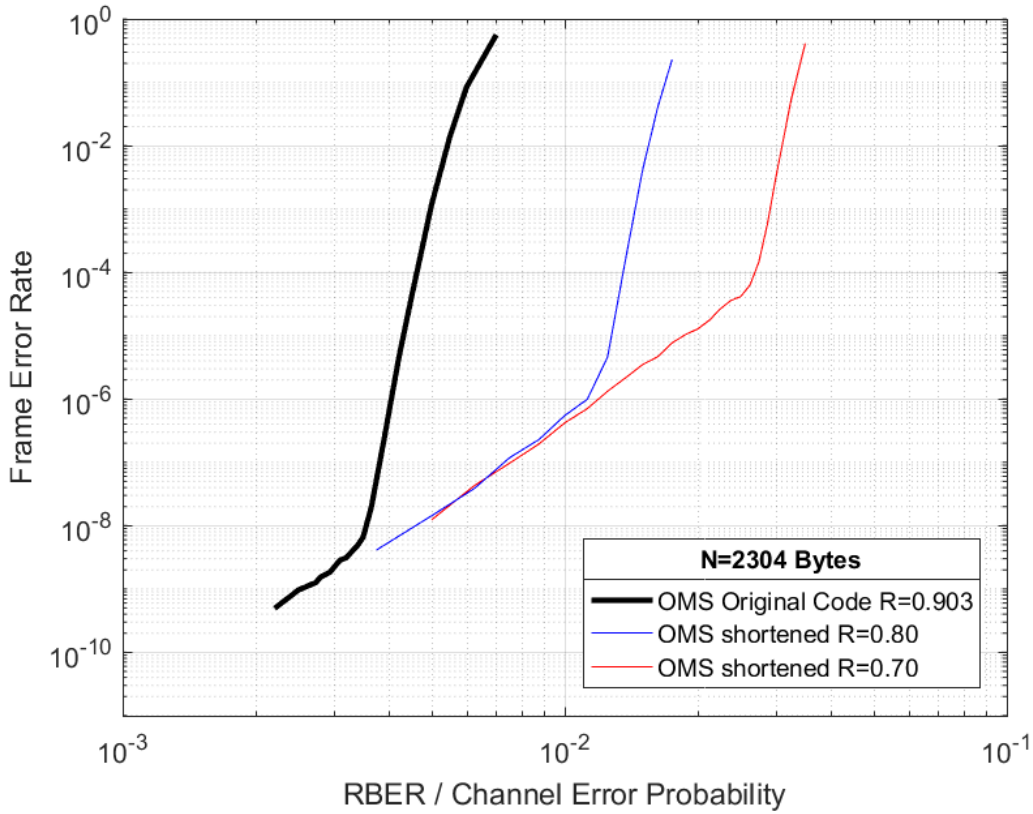
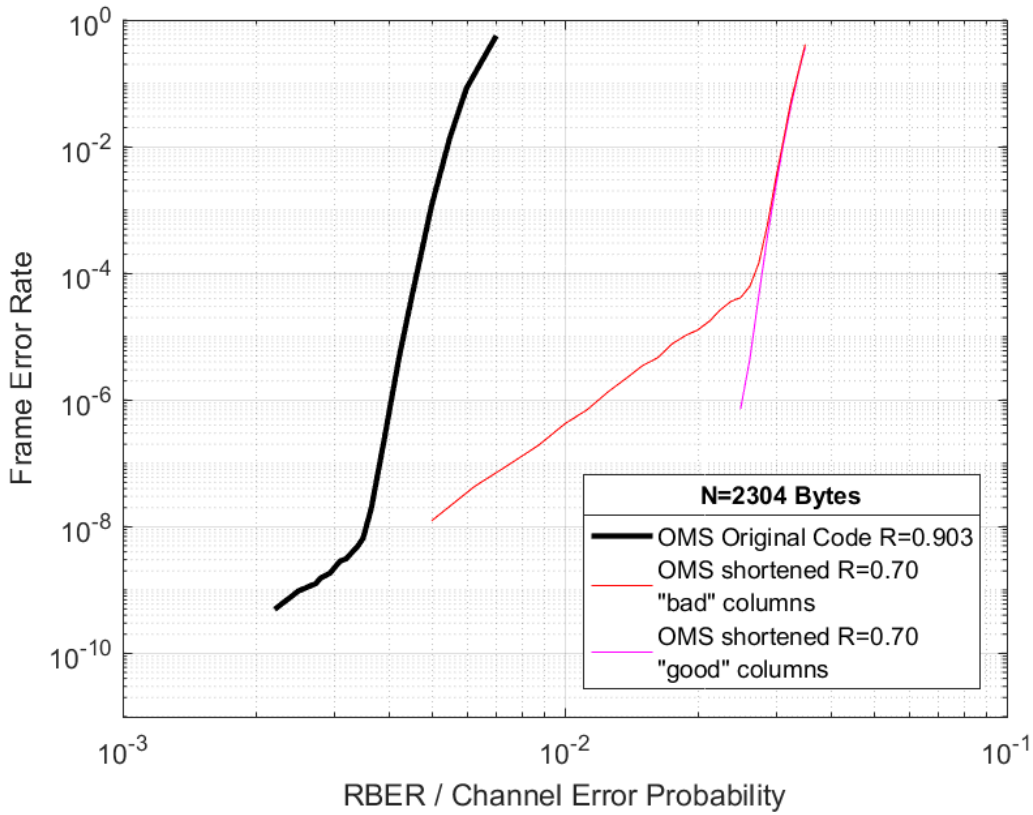


Selecting the correct block-columns is critical

Target QC-LDPC code : N=2kBytes - $R = 0.903$ - $(M_b, N_b) = (56, 576)$ - $L = 32$

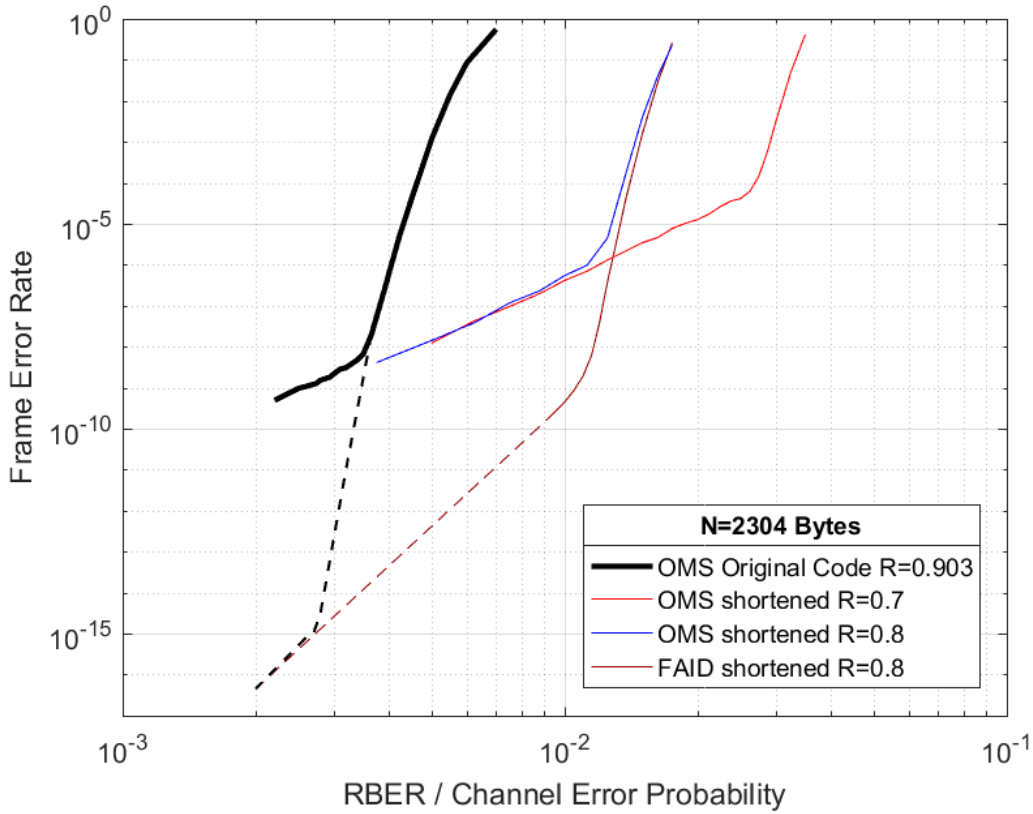
- "Bad" Shortened Code C_2 : $(M_b, N_b) = (56, 182)$ - $R = 0.692$
- "Good" Shortened Code C_2^{bis} : $(M_b, N_b) = (56, 182)$ - $R = 0.692$
- **Decoder :** Offset corrected Min-Sum / 4 bits precision / 20 iterations



Results with FAID decoders

Finite Alphabet Iterative Decoders (FAID) have an inherent error-floor mitigation property

- Our prediction method is valid for ANY decoder.
- The prediction of the Error Floor shows that FAID gains **5 orders of magnitude** compared to Offset-Min-Sum.



Outline

Conclusion

What we have shown

- A computationally efficient method for estimating error floor of QC-LDPC codes over the BSC channel for arbitrary message update rules,
- Applicable to regular and irregular codes, and for more general binary-input memoryless channels
- Graphs of small expansion in the Tanner graph are exhaustively expanded and contracted to obtain subgraphs that are true harmful trapping sets,
- Based on harmfulness of trapping sets, the code is shortened but in a way that it still contains the most harmful trapping sets,
- Allows to predict **very deep error floors**,

Way forward

- Validate our method for irregular codes, and for the BI-AWGN channel,
- Use the new harmfulness characterization to design LDPC codes with very low error-floor.