# Random-Access LDPC Codes

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***Abstract*—New types of LDPC codes motivated by practical storage applications are presented. LDPCL codes (suffix 'L' stands for locality) can be decoded locally at the level of sub-blocks that are much smaller than the full code block, thus providing fast random access to the coded information. The same code can also be decoded globally using the entire code block (as usual), for increased data reliability. We present constructions of LDPCL and spatially-coupled (SC) LDPCL codes that enable random access, and we exemplify their benefits over ordinary LDPC codes.**

## 1. Introduction

The growing demand for denser storage devices results in an increase in their error rates. Since in storage every decoding failure implies data loss, storage applications require very strong error-correcting codes (ECCs). Low-density parity-check (LDPC) codes and spatially-coupled (SC) LDPC codes with their low-complexity iterative decoding algorithm *Belief-Propagation* are powerful methods to achieve this storage reliability with high rates. Another key feature for modern storage devices is fast access, i.e., low-latency high-throughput read operations with moderate block sizes. This feature conflicts with the stringent reliability requirements in devices.

This inherent conflict motivates a coding scheme that enables fast read access to small (sub) blocks with modest data protection and low complexity. In case of a decoding failure, a high data-protection "safety net" in the form of a stronger code with longer block is decoded. We call these codes *"random-access"* codes, in the sense that small sub-blocks of data can be accessed independently. In this paper we present constructions and decoding strategies for random-access LDPC and SC-LDPC codes.

In random-access codes (also called multi-block codes [1]), code-blocks of length $N$ are partitioned into $M$ sub-blocks with $n$ coded symbols each (i.e., $N = Mn$). Each sub-block is a codeword of some code, and can be locally decoded, independent from other sub-blocks, to extract $k$ information symbols. Local decoding is fast and provides error-correcting capabilities that are lower than the worst-case reliability requirements. In case of catastrophic error events, the whole code-block is accessed to provide the data-reliability needed, and $K = Mk$ information symbols are decoded. For a wider and deeper study of random-access LDPC codes, please refer to [6] and [7].

## 2. LDPCL Codes

Every LDPC code can be represented by a bipartite graph (Tanner graph) with nodes partitioned to variable nodes (VNs) and check nodes (CNs). We LDPCL codes of length $N = Mn$ through multi-block Tanner graph. In these graphs, the VNs are divided to $M$ disjoint sets of size $n$ each, and the CNs are divided into two sets: *local CNs* and *joint CNs*. The graph construction is constrained such that each local CN is connected only to VNs that are in the same sub-block; this constraint ensures sub-block decoding. The joint CN connections have no constraints. We denote by $\lambda_{L,i}$ (resp. $\rho_{L,i}$) the fraction of local edges connected to VNs (resp. CNs) with local-degree (resp. degree) $i$. We call $\{\lambda_{L,i}, \rho_{L,i}\}$ local-degree distributions. The joint-degree distributions $\{\lambda_{J,i}, \rho_{J,i}\}$ are defined similarly but with an important difference: we allow VNs to have joint degrees 0 or 1. This turns out to be necessary to increase the code rate. We use $P_0$ to denote the fraction of jointly-unconnected VNs. The parameters $M$, $n$, $\lambda_L, \lambda_J, \rho_L, \rho_L, P_0$ define a family of LDPCL codes, with a design rate given by $R = 1 - \frac{\int_0^1 \rho_L}{\int_0^1 \lambda_L} - \frac{\int_0^1 \rho_J}{\int_0^1 \lambda_J}(1-P_0)$ ,. Here we write $\lambda_L$ (or any other degree distribution) as a notation for the generating polynomial $\lambda_L(x) = \sum_i \lambda_{L,i} x^{i-1}$.

We suggest a decoding algorithm that operate in two modes: *local mode* and *global mode*. In the local mode, the decoder tries to decode a sub-block of length $n$ using the corresponding local sub-graph. If the decoder meets a failure criterion, it enters the global mode where it decodes the entire code block using the complete multi-block Tanner graph.

Figure 1 compares the BER of three rate-half codes over the additive Gaussian channel with the variance $\sigma$ as a free parameter: 1) an ordinary LDPC code with degree distributions $\lambda(x) = 0.27684x + 0.28342x^2 + 0.43974x^8$, $\rho(x) = 0.01568x^5 + 0.85244x^6 + 0.13188x^7$, and block length $n = 4KB$; 2) an ordinary LDPC code with same degree distributions, but shorter block length $n = 1KB$ (the degree distributions are from [8]); 3) a random-access LDPCL code with $M = 4$ sub-blocks of length $n = 1KB$ each, local degree distributions $\lambda_L(x) = 0.1575x + 0.3429x^2 + 0.0363x^5 + 0.059x^6 + 0.279x^8 + 0.1253x^9$, $\rho_L(x) = 0.8266x^{34} + 0.1345x^{35} + 0.0087x^{70} + 0.0302x^{71}$, joint degree distributions $\lambda_J, \rho_J$ which are equal to those of the first two codes, and $P_0 = 0.22$. The asymptotic theoretical threshold for all three codes is $\sigma^* = 0.9497$. The random-access LDPCL code also enables sub-block decoding with an asymptotic threshold $\sigma_L^* = 0.5$. The plot exemplifies the benefits of LDPCL codes: in one hand, the LDPCL code provides local access to 1KB sub-blocks unlike the LDPC 4KB code, and on the other hand, the LDPCL global data-reliability is better than the LDPC 1KB code.

## 3. SC-LDPCL Codes

SC-LDPC codes [2] are LDPC codes in which the sparse parity-check matrix has a band-diagonal structure. SC-LDPC

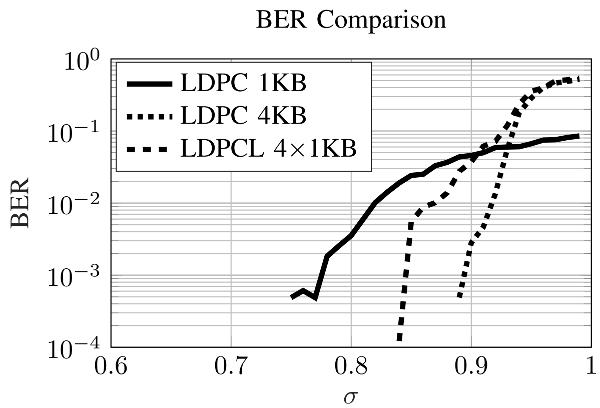


Fig. 1. BER comparison between three rate-half codes: an LDPC code with block length $n = 4KB$; an LDPC code with same degree distributions, but shorter block length $n = 1KB$; a random-access LDPCL code with $M = 4$ sub-blocks with length $n = 1KB$ each.

codes were shown to have many desired properties. For example, threshold saturation [3], linear-growth of minimum distance [4] and linear-growth of minimal trapping sets of typical codes from the ensemble [5].

One way to construct a SC-LDPC code is via protographs. A protograph is a small bipartite graph $\mathcal{G} = (\mathcal{V} \cup \mathcal{C}, \mathcal{E})$, where $\mathcal{V}, \mathcal{C}$ and $\mathcal{E}$ are the sets of VNs, CNs and edges, respectively. A Tanner graph is generated from a protograph $\mathcal{G}$ by a lifting operation ("copy-and-permute") specified by a parameter $L$ (see [4]). Let $1^{l\times r}$ be an all-ones bi-adjacency matrix with $l$ rows and $r$ columns representing an $(l, r)$-regular protograph. An $(l, r)$-regular SC-LDPC protograph is constructed by diagonally placing copies of $\big(B_1; B_2\big)$, where $B_1, B_2$ are non-negative $l \times r$ matrices such that $B_1 + B_2 = 1^{l\times r}$ (the results readily extend to coupling more matrices). For example, the spatially-coupled $(3, 6)$ protograph is generated by $B_1 = \big(v_1; v_2; v_3\big)$, where $v_1 = (1, 1, 0, 0, 0, 0)$, $v_2 = (1, 1, 1, 1, 0, 0)$, $v_3 = (1, 1, 1, 1, 1, 1)$.

To the best of our knowledge, existing constructions of SC-LDPC codes do not enable sub-block access (for example, the $(l, r)$ SC-LDPC ensemble from [4, Definition 3] with $l = \gcd(l, r)$). This motivates a construction of $(l, r)$-regular random-access SC-LDPCL codes [7]:

**Construction 1.** *Let $t \in \{1, 2, \ldots, l-2\}$, and let $A_1$ be the $t \times r$ matrix illustrated in Figure 2 (a), where $\underline{1}$ and $\underline{0}$ are length-$\lfloor \frac{r}{t+1} \rfloor$ all-one vector and length-$\Big(r - t\lfloor \frac{r}{t+1} \rfloor\Big)$ all-zero vector, respectively. Let $A_2$ be an all-ones $(l-t) \times r$ matrix. We build the $(l, r, t)$ protograph $M$ copies of $\big(B_1; B_2\big)$ on the diagonal, where $B_1 = \big(A_1; A_2\big)$ and $B_2 = 1^{l\times r} - B_1$.*

We call the parameter $1 \le t \le l-1$ *the coupling parameter*. It serves as a design tool that controls the trade-off between the local and global decoding capabilities: small values of $t$ implies high local noise resilience, and large values of $t$ leads to high global data protection. Figure 2 (b) illustrates the $(l = 3, r = 6, t = 1)$ SC-LDPCL protograph with $M = 3$ SBs. Each sub-block in this protograph can be decoded independently to the others.

SC-LDPCL codes can be decoded in a mode in between local and global decoding, which we call *semi-global (SG) decoding*. SG decoding has a substantial complexity advantage over global decoding with a very small cost in data-protection. Consider a SC-LDPCL protograph with $M$ SBs, and assume that the user wants to extract SB $m \in [1 : M]$. We call SB $m$ the *target*. In SG decoding, the decoder uses $d$ *helper* SBs to decode the target in two phases: the *helper phase*, and the *target phase*. In the former, helper SBs are decoded locally using information from other previously-decoded helper SBs, and in the latter, the target SB is decoded using information from its neighboring helper SBs. Figure 3 exemplifies SG decoding with $d = 4$ helper SBs. The helper phase consists of decoding helpers $m-2$ and $m+2$ locally, and decoding helpers $m-1$ and $m+1$ using the information from helpers $m-2$ and $m+2$ , respectively. In the target phase, SB $m$ is decoded using information from both SB $m-1$ and $m+1$.

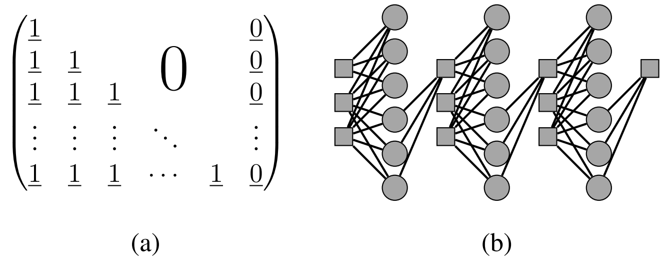


Fig. 2. (a) Matrix $A_1$ from Construction 1. (b) The $(3, 6, 1)$ SC-LDPCL protograph with $M = 3$ SBs.

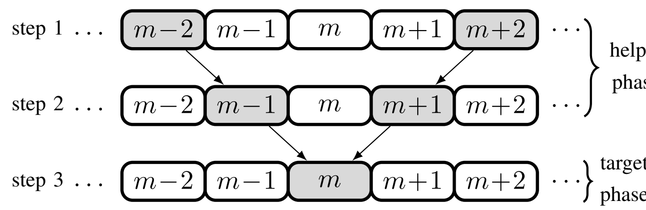


Fig. 3. Example of SG decoding with target SB $m \in [1 : M]$, and $d = 4$; the steps are shown from top to bottom. The gray SBs are those that are decoded in a given step, and the arrows represent information passed between sub-blocks.

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