Polar Coding for Selector-less Resistive Memories

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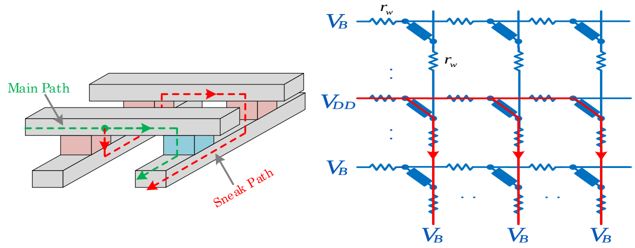
***Abstract*— Transistor-based memories are rapidly approaching their maximum density per unit area. Resistive crossbar arrays enable denser memory due to the small size of switching devices. However, due to the resistive nature of these memories, they suffer from current sneak paths complicating the readout procedure. In this work, we propose an error-correcting scheme mitigating the sneak path effect based on polar codes. We describe the proposed code construction and show numerically the performance improvement in terms of bit error rate.**

I. Introduction

Emerging nonvolatile memories (NVMs), such as phase change memory (PCRAM), ferroelectric memory (FeRAM), spin transfer torque magnetic memory (STT-MRAM), and resistive memory (RRAM), have shown high potential as alternatives for floating-gate-based nonvolatile memories [1]. RRAMs are considered the best candidate for the next generation nonvolatile memory due to their high reliability, fast access speed, multilevel capabilities and stack-ability creating 3D memory architectures. To achieve higher density memories, switching devices are sandwiched between the crossbar metal layers, without using access devices such as transistors, diodes and selectors. The main disadvantage of the selector-less (gate-less) crossbar-based memories is the sneak path effects which limit the readability of the array.

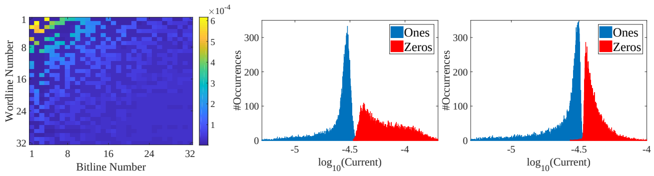
The sneak path problem arises because of two reasons: a) The existence of many paths from the inputs to the outputs as illustrated in Fig. 1a, which shows the sneak path in a 2×2 crossbar array. The sneak path currents highly disturb the desired current which is proportional to the stored resistance. This can be ideally eliminated by parallel reading of the entire row, where all the columns (bitlines) are grounded to absorb the current as illustrated in Fig. 1b. b) The existence of wire resistance is inevitable in nano-crossbar arrays and can reach up to 90Ω per cell for nano feature sizes (i.e.$F = 5nm$) [2]. The wire resistance creates voltage drops, which are functions of the stored data and accumulate throughout the array. Fig. 2a shows the measured current of each cell in a 32×32 array with 25Ω wire resistance, storing random data. Clearly, the sensed current of low resistance state decreases in both vertical an horizontal directions in the array. The top left cells have less disturbed behavior and stored ones and zeros are distinguishable. On the other hand, the bits of the right-bottom cells are indistinguishable due the read margin overlap. Fig. 2b and 2c show the histogram of the measured currents of the 16^{th} and the 32^{nd} bitlines. Clearly, the larger the bitline index is, the more errors occur.

In this work, we design polar codes for resistive memory cells with varying error probabilities due to wire resistance,



(a) (b)

Fig. 1: Crossbar array with the sneak path problem (a) one port reading, and (b) parallel reading technique.



(a) (b) (c)

Fig. 2: (a) Measured current per cell, and its histogram for bitline number (b) 16, and (c) 32.

and demonstrate the performance improvement. Polar codes [3] are the first family of explicit error-correcting codes to provably achieve the capacity of binary symmetric channels, with a low-complexity encoding and successive cancellation decoding (SCD). For a code of length N, encoding/decoding has a complexity of $O(N \log N)$. For the above reasons, polar codes constitute an attractive error correction scheme.

II. Non-stationary Polar Codes Construction

Polar codes [3] manufacture out of N independent copies of a given binary discrete memory-less channel, a second set of N *synthesized* binary-input channels. The channels show a polarization effect, namely, as N becomes large, the symmetric capacities of the synthesized channels tend towards 0 or 1 for all but a vanishing fraction.

In our framework, in contrast to the original polar codes, we consider the extension polar codes to the setting where the underlying channels are of *varying reliability levels* (see Fig. 3). Using the terminology in [4], we refer to such polar codes as non-stationary polar codes.

1) Polar code encoding and decoding: Let $\vec{u}_N = (u_0, u_1, \ldots, u_{N-1})$ and $\vec{x}_N = (x_0, x_1, \ldots, x_{N-1})$ be respectively the input and the output of a length-N polar code, $N = 2^n$ for some integer n. The encoding of polar codes is given by $\vec{x}_N = \vec{u}_N G_N$, where $G_N = [1\,0; 1\,1]^{\otimes n}$ is constructed using the Kronecker power. The original i^{th} channel is denoted by W_i, and its symmetric capacity is $I(W_i)$, $1 \leq i \leq N$. After polar encoding, one obtain N

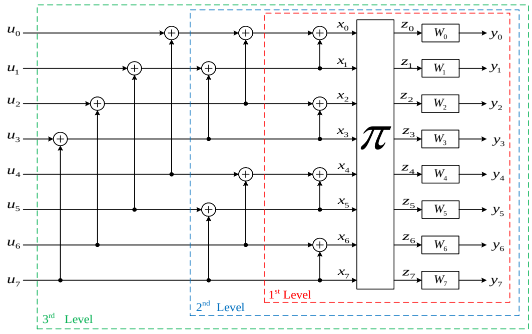


Fig. 3: Polar encoding with $N = 8$ channels, with the permutation π applied at the 1^{st} level of polarization.

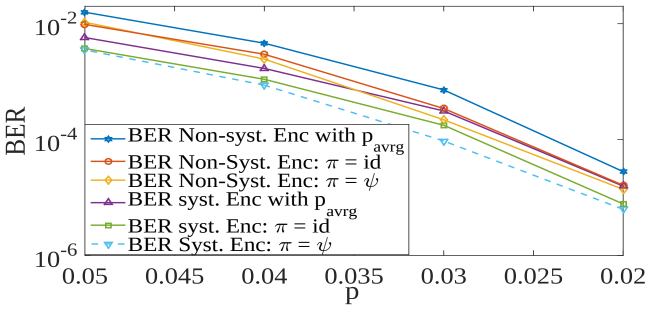


Fig. 4: Performance evaluation. $N = 256, k = 128$.

synthesized channels. We denote by $I(W^{(i)})$ the symmetric capacity of the i^{th} synthesized channel, with input u_i. A polar code of dimension k transmits k information bits in the k synthesized channels with the highest $I(W^{(i)})$ (we denote the corresponding information set by $\mathcal{I}$), and $N-k$ arbitrary but fixed bits in the remaining $N-k$ synthesized channels. Decoding of polar codes is carried out using SCD as in [3].

2) Non-stationary polar coding: We assume w.l.o.g that the original channels have increasing symmetric capacity, i.e., $0 < I(W_1) \leq \ldots \leq I(W_N) < 1$. The performance of polar codes depends on the synthesized channels of the information set $\sum_{i\in\mathcal{I}} I(W^{(i)})$ [3]. As illustrated in Fig. 3, for non-stationary channels, we seek to apply a permutation π to the vector $\vec{x}_N$ in order to enhance the overall performance. Ideally, we want to use a permutation π such that $\sum_{i\in\mathcal{I}_{\pi^*}} I(W_{\pi^*}^{(i)}) \geq \sum_{i\in\mathcal{I}_\pi} I(W_\pi^{(i)})$.

In our construction, we choose $\pi = \psi$, the *bit-reversal permutation*, which reverses the n binary bits of each index between 0 and $N-1$. For the toy example of $N = 4$ binary erasure channels consisting of two types of channels, ψ can be proven to be optimal for any target rate.

Example 1: We consider N binary symmetric channels (BSC) with varying cross-over probabilities, linearly spaced and centered at value p, with maximum deviation of .0075. We also evaluate the performance of a regular polar code designed for the *average* BSC channel, with p_{avrg} satisfying $N(1-h(p_{avrg})) = \sum_{i=0}^{N}(1-h(p_i))$, where $h(p)$ is the binary entropy function. We compare the bit error rate (BER) performance with systematic and non-systematic encoding, for 10^5 runs, in Fig. 4. The regular polar code exhibits the worst BER, while the non-stationary polar code with $\pi = \psi$ performs the best.

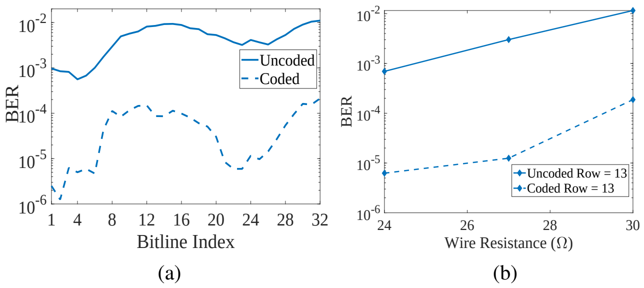


Fig. 5: Uncoded vs. Coded BER performance.

III. Simulation Results

We apply non-stationary polar codes with $\pi = \psi$ to mitigate the sneak path problem in the crossbar array. We use SPICE-like resistive crossbar simulator for the resistive memory readout current, proposed in [2]. Specifically, we first estimate a single detection threshold for each wordline so as to minimize the overall uncoded BER per word. This transforms the read channels into BSC's. The threshold for each wordline is estimated by generating large training data and then applying a good binary classifier. For instance, we observe that logistic regression-based classifier gives superior performance for most parameters. For a crossbar array of size $m \times n$, there are m wordlines, each with n cells (channels) of varying crossover probabilities. We finally encode each word using non-stationary polar codes with $\pi = \psi$, with the appropriate parameters for each wordline.

Example 2: We consider a 32×32 array. In Fig. 5a, we simulate the BER over 2×10^5 runs for 27Ω wire resistance and $1/2$ coding rate. Clearly, the BER performance of the coded data is much better than uncoded data (a smoother coded BER curve can be obtained with higher simulation runs). In Fig. 5b, we visualize the BER performance for one particular row for three wire resistance values. We observe that coded and uncoded BER performance degrades as the wire resistance increases. This is expected as the sneak path effect increases causing more errors.

We note that the performance of polar codes can be enhanced by using more sophisticated decoding algorithms, at the expense of higher complexity. Moreover, by the sneak path nature, errors in neighboring cells are not independent. A future direction of research is to investigate enhanced polar decoding algorithms taking into consideration the sneak path nature.

References

[1] D. C. Daly, L. C. Fujino, and K. C. Smith, "Through the looking glass-the 2017 edition: Trends in solid-state circuits from isscc," *IEEE Solid-State Circuits Magazine*, vol. 9, no. 1, pp. 12–22, 2017.

[2] M. E. Fouda and et.al, "Modeling and analysis of passive switching crossbar arrays," *IEEE Trans. on Circuits and Systems I: Regular Papers*, vol. 65, no. 1, pp. 270–282, 2018.

[3] E. Arikan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3051–3073, 2009.

[4] M. Alsan and E. Telatar, "A simple proof of polarization and polarization for non-stationary memoryless channels," *IEEE Trans. Inf. Theory*, vol. 62, no. 9, pp. 4873–4878, 2016.