Optimal Data Shaping Code Design

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Outline

Introduction

- Flash memory: the most widely used non-volatile memory
  - fast read/write speed
  - low power consumption
- Flash memory cells gradually wear out during program-erase (P/E) cycling.
- Damage from programming the cell depends on the cell level. Programming a cell to higher level induces more damage.
- Enhancing lifetime by using shaping codes
  - Endurance code[1]: shapes random (unstructured) data with a given rate
  - Direct shaping code[2,3] : shapes structured data with rate 1

[1] A. Jagmohan, M. Franceschini, L. A. Lastras-Montano and J. Karidis, "Adaptive endurance coding for NAND Flash," *2010 IEEE Globecom Workshops*, Miami, FL, 2010, pp. 1841-1845.

[2] E. Sharon, et al., Data Shaping for Improving Endurance and Reliability in Sub-20nm NAND, presented at Flash Memory Summit, Santa Clara, CA, August 4-7, 2014.

[3] Y. Liu and P. H. Siegel, "Shaping codes for structured data," in *Proc. IEEE Globecom*, Washington, D.C., Dec. 4-8, 2016, pp. 1–5.

Definition of General Shaping Codes

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- Let $\mathbf{X} = X_1 X_2 \ldots$ be an i.i.d source with alphabet $\mathcal{X} = \{\alpha_1, \ldots, \alpha_u\}$. The distribution of $\mathbf{X}$ will be denoted by P $(P_1 \geq P_2 \geq \ldots P_u)$.

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- Let $\mathcal{Y} = \{\beta_1, \ldots, \beta_v\}$ be an alphabet and $\mathcal{Y}^*$ the set of all finite sequences over $\mathcal{Y}$, including the null string λ of length 0. Every β_i corresponds to a cost U_i $(U_1 \leq U_2 \leq \ldots \leq U_v)$.

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A shaping code is defined as a prefix-free mapping $\phi : \mathcal{X}^q \to \mathcal{Y}^*$ which maps x_1^q to a variable length sequence y^*.

Example

- Input: $\mathcal{X} = \{0,1\}$, $X \sim Ber(\frac{1}{2})$
- Outut: $\mathcal{Y} = \{0,1\}$, $U_0 = 0.585$ and $U_1 = 1.585$
- Shaping code defined by mapping $\{11 \to 111, 10 \to 110, 01 \to 10, 00 \to 0\}$.

Expansion Factor

Definition

- The expected length of a codeword is

$$E(L) = \sum_{x_{\mathbf{1}}^{q} \in \mathcal{X}^{q}} P(x_1^q) L(\phi(x_1^q)). \tag{1}$$

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Example

- Shaping code defined by mapping $\{11 \to 111, 10 \to 10, 01 \to 10, 00 \to 0\}$.
- $E(L) = \frac{1}{4}(3 + 3 + 2 + 1) = 2.25$
- $f = \frac{E(L)}{q} = 1.125$

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The probability of occurrence $\hat{Y}$ in encoded sequences $\phi(\mathbf{X})$ is

$$\hat{P}_i = Pr(\hat{Y} = \beta_i) = \lim_{l \to \infty} \sum_{y_1^l} N_i(y_1^l) Q(y_1^l)/l = \lim_{l \to \infty} \frac{E(N_i(Y_1^l))}{l}. \tag{3}$$

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Lemma

For a prefix-free shaping code $\phi : \mathcal{X}^q \to \mathcal{Y}^*$, $\hat{Y}$ exists and

$$\hat{P}_i = E(N_i(\phi(X^q)))\frac{1}{E(L)} \tag{4}$$

Once we know the probability of occurrence, we can calculate the cost per output symbol $\sum_i \hat{P}_i U_i$ (we also call it average wear cost).

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$$\begin{aligned} \underset{\hat{P}_i}{\text{minimize}} \quad & \sum_i \hat{P}_i U_i \\ \text{subject to} \quad & H(\hat{Y}) \geq \frac{H(\mathbf{X})}{f} \\ & \sum_i \hat{P}_i = 1. \end{aligned} \tag{5}$$

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- High rate is required in flash memory device for low encoding/decoding time complexity and high storage capacity.

Type-I and Type-II Minimization

- The second goal is to minimize the average cost per input symbol (total cost) and find the optimal expansion factor (Type-II minimization).

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Performance of Shaping Code

Theorem (Optimal Type-I Shaping)

Given the distribution P of source words and a cost vector $\mathcal{U}$, the minimum average wear cost we can get from a shaping code $\phi : \mathcal{X}^q \to \mathcal{Y}^*$ with expansion factor $f = \frac{E(L)}{q}$ is bounded by $\sum_i \hat{P}_i U_i$, where $\hat{P}_i = \frac{1}{N} 2^{-\mu U_i}$, μ is a positive constant selected such that $H(\hat{Y}) = \sum_i -\hat{P}_i \log_2 \hat{P}_i = H(\mathbf{X})/f$, and N is a normalization constant.

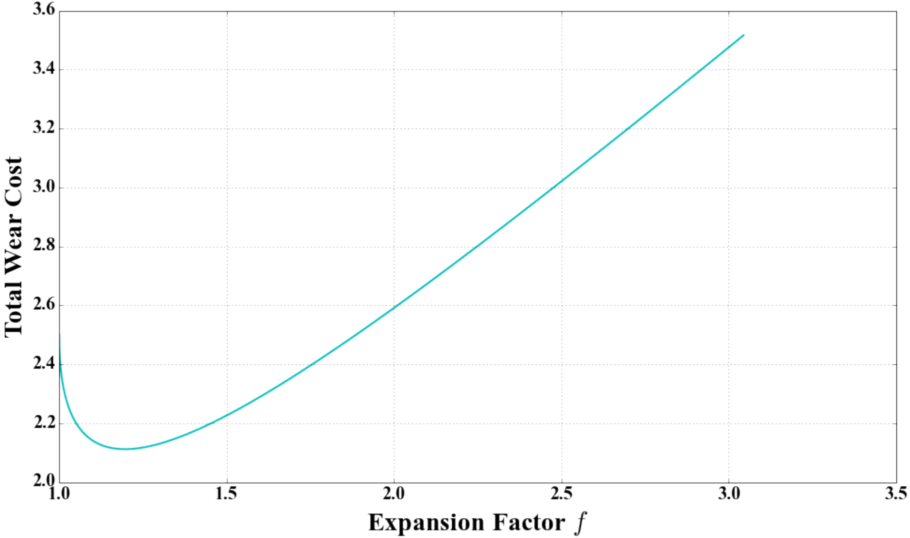
Theorem (Optimal Type-II Shaping)

Let P be the source distribution and let $\mathcal{U}$ be a cost vector. If $U_1 \neq 0$, then the minimum total wear cost of a shaping code $\phi : \mathcal{X}^q \to \mathcal{Y}^*$ is given by $f \sum_i \hat{P}_i U_i$, where $\hat{P}_i = 2^{-\mu U_i}$, μ is a positive constant selected such that $\sum_i 2^{-\mu U_i} = 1$, and the expansion factor f is

$$f = \frac{H(\mathbf{X})}{-\sum_i \hat{P}_i \log_2 \hat{P}_i}. \tag{7}$$

If $U_1 = 0$, then the total cost is a decreasing function of f. □

Minimal total wear cost vs expansion factor f when source is random with cost [1,2,3,4]



Equivalence Theorem and Separation Theorem

Theorem (Equivalence Theorem)

Let P be the source distribution. A type-I shaping code with cost vector $\mathcal{U}$ and expansion factor f is a type-II shaping code with cost vector $\mathcal{U}'$ where

$$U_i' = -\log_2 \hat{p}_i. \tag{8}$$

$\hat{P} = \{\hat{p}_i\}$ is the optimal probability distribution given in optimal type-I shaping theorem.

Theorem (Separation Theorem)

An optimal general shaping code for a given expansion factor f can be constructed by a concatenation of lossless compression with type-II shaping code for uniform i.i.d source.

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Theorem (Separation Theorem)

An optimal general shaping code for a given expansion factor f can be constructed by a concatenation of lossless compression with type-II shaping code for uniform i.i.d source.

- Equivalence theorem: There is a bijection between optimal type-I and optimal type-II shaping codes.
- Separation theorem: We only need to design shaping code for uniform i.i.d source.

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Require: Source $\mathbf{X}$, cost vector $\mathcal{U}$, expansion factor f

1: Compress the source file, calculate the compression ratio g, for a optimal lossless compression, $g = \frac{\log_2 |\mathcal{X}|}{H(\mathbf{X})}$. Set $f' = fg$.

2: Calculate symbol probability distribution $\hat{P} = \{\hat{p}_i\}$ minimizing average cost for a uniform random source and expansion factor f' using optimal type-I shaping theorem.

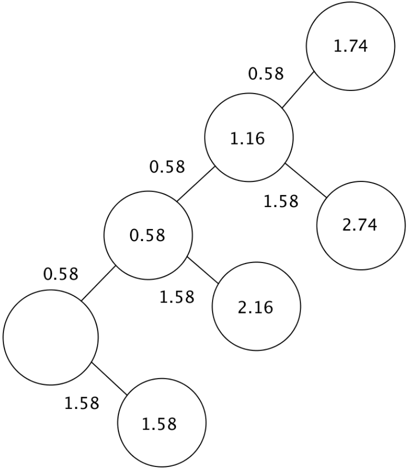
3: Define a cost vector $\mathcal{U}' = \{U_i'\}$ by $U_i' = -\log_2 \hat{p}_i$.

4: Design a type-II shaping code for a uniform i.i.d source and cost vector $\mathcal{U}'$.

5: Concatenate an optimal lossless compression code with the shaping code designed in the preceding step.

Optimal Shaping Code Design

- Type-II shaping: Varn Codes.
- Tree-based, fixed-to-variable length codes that minimize total cost for a specified codebook size K.
- Designed specifically for uniformly distributed i.i.d source.
- Expand the leaf node that has the minimum cost.
- Example: symbol '1' has cost 0.58, symbol '0' has cost 1.58, codebook size $N = 4$ ($q = 2$).



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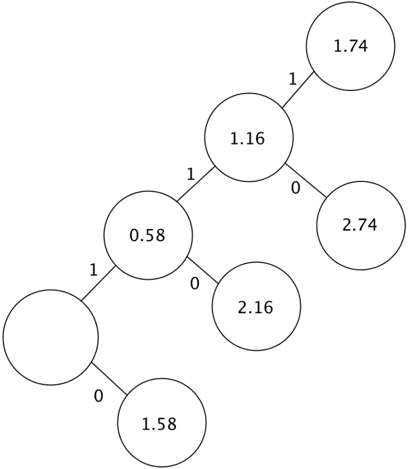

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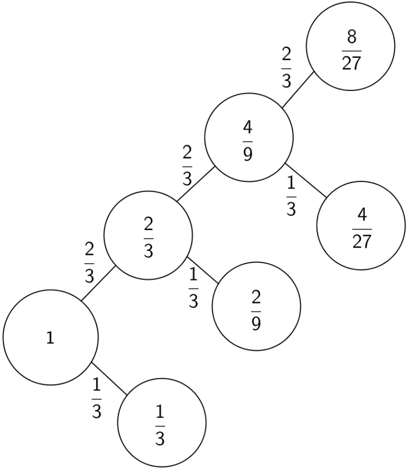

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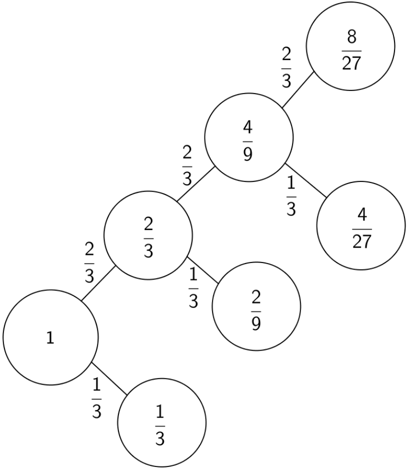
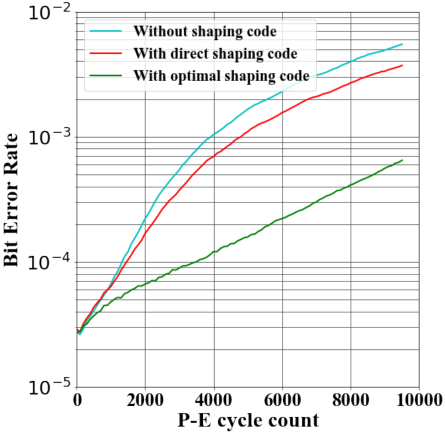


Experiment Setup

- MLC shaping code was applied to the ASCII representation of the English-language novel *The Count of Monte Cristo* and Chinese-language work *Collected Works of Lu Xun, Volumes 1–4*.
- The original file and file coded with rate-1 type-I shaping code were written to our flash memory testboard.
- The first half of the data was written on the lower page and the second half of the data was written on the upper page.
- For the next programming cycle, we "rotate" the data. The data written on the i-th wordline is written on the (i+1)-st wordline.
- After every 100 cycles, pseudo-random data is written to the block and then read back to calculate the bit-error-rate (BER).
- To compare the performance of shaping code with compression, we rescaled the P/E cycle count of the shaping code by the compression ratio and compared the result to P/E cycling of pseudo-random data.

Bit Error Rate Results



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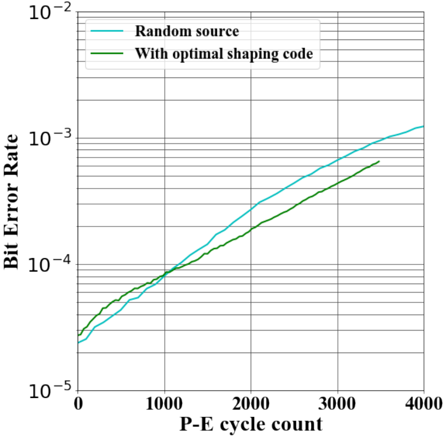

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BER Performance of English-language novel

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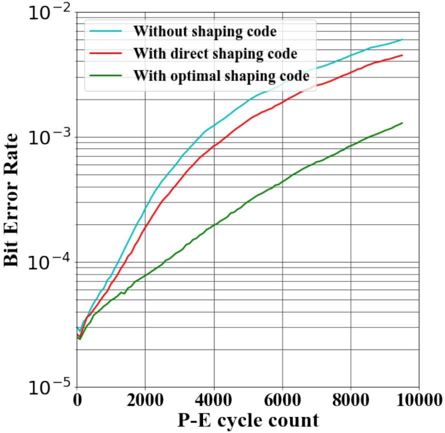

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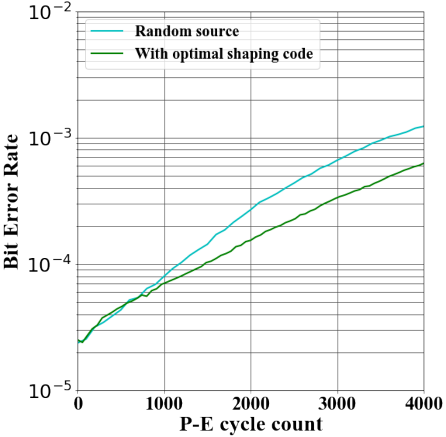

(b)

BER Performance of Chinese-language novel

Conclusion

- Shaping code is used to reduce the average wear cost and total wear cost.
  - Type-I shaping: minimize cost per output symbol.
  - Type-II shaping: minimize cost per input symbol.
- Equivalence theorem and separation theorem suggest how to design the shaping code encoder.
  - Type-I shaping: convert this problem into a type-II shaping problem.
  - Type-II shaping: convert this problem into a concatenation of compression and type-II shaping for uniform i.i.d source.
- Optimal type-II shaping codes for a uniform i.i.d source: Varn Codes.
- Experimental results for MLC shaping codes on English and Chinese text show a reduction in bit error rate.